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LASER DOPPLER ANEMOMETER MEASUREMENT AND ANALYTICAL COMPARISON OF FLOW AROUND A CYLINDER AT LOW REYNOLDS NUMBER

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Monterey, California

THESIS

Laser Doppler Anemometer Measurement and Analytical Comparison of Flow Around a Cylinder at Low Reynolds Number

by

Terry Scott Wanner

March 1976

Thesis Advisor: D. J. Collins

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Laser Doppler Anemometer Measurement and Analytical Comparison of Flow Around a Cylinder at Low Reynolds Number

by

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B.S., United States Naval Academy, 1969

Submitted in partial fulfillment of the
requirements for the degree of

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I. ABSTRACT

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A finite element grid was constructed using triangular elements which encompassed the upper half-cylinder. Flow measurements using the laser velocimeter were confined to this region of interest.

Theory was compared against experimental results, and the feasibility of the utilization of this particular type of laser anemometer was evaluated.

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II. DESCRIPTION OF LASER ANEMOMETER

A basic laser anemometer is comprised of four primary elements:

(1) a coherent light source

(2) optics

(3) signal converter (photomultiplier tube)

(4) signal processor/display

The principle of the laser anemometer is to create a grid pattern and measure the speed with which a particle in a fluid free stream passes through the grid. The following assumptions were taken into consideration:

(1) particles injected into the free stream are small enough to faithfully follow stream lines of the flow, thus by measuring particle characteristics, we are actually measuring the medium's characteristics. This is easily achieved by use of particles which are on the order of a few microns in diameter.

(2) particle generation results in a consistent particle size, or at least a majority of the particles are within a specified range of sizes. For example, DOP (di-2-ethylhexyl phthalate) particle generators produce particles with a mean diameter of 0.75 microns and burning turbine oil produces particles on the order of 4-6 microns (from electron microscope).

A laser is incorporated because of its coherency and power. The aforementioned grid is in reality the result of two coherent beams interfering with themselves (Fig. 1). The optics produce the beam pair by conventional splitting and focusing. Because the laser beam intensity is of a Gaussian distribution the grid created by constructive and destructive interference is not uniform over the entire focal volume. The fringes created on the outer porticns of the horizontal axis are not perfectly formed because of slightly different intensities in that area. The main portion of the interference grid is uniform and, most importantly, the fringe spacing (D_f) is determined only by the laser wavelength (lambda) and the half angle of the beam pair convergence (theta/2).

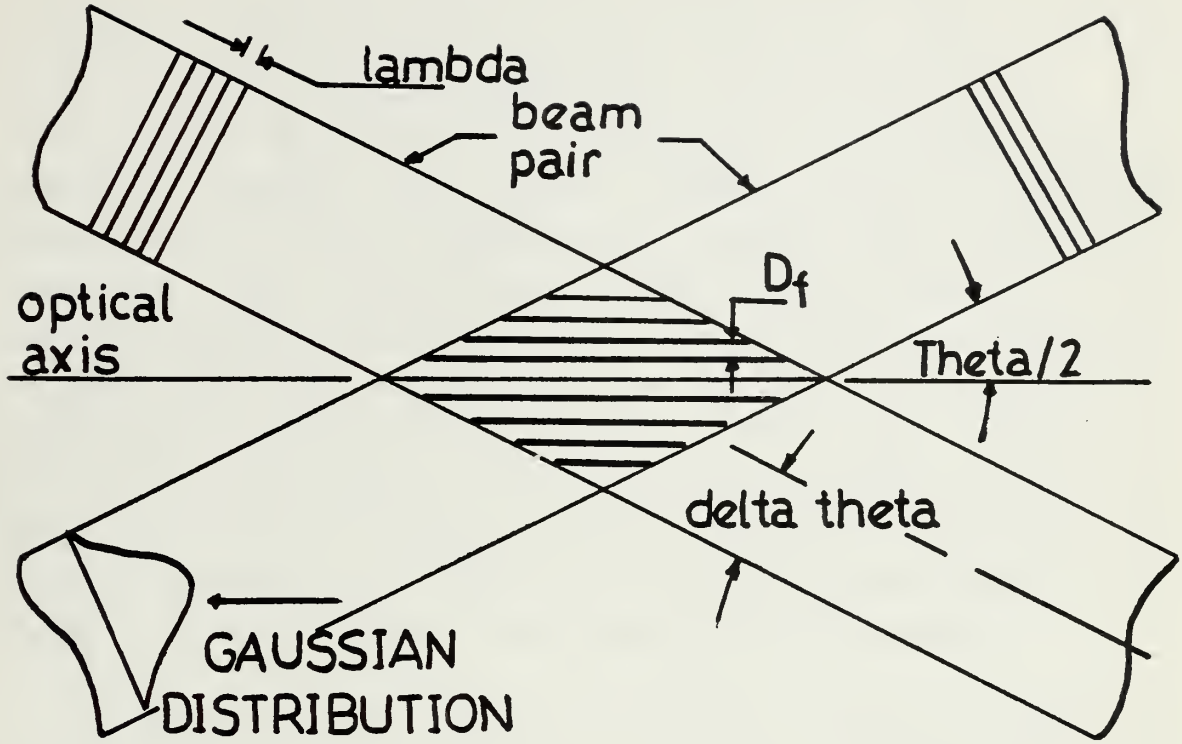
$$D_f = \text{lambda}/(2*\sin(\text{Theta}/2))$$

The number of fringes (N_f) may also be computed, provided the beam focusing convergence angle (delta theta) is known ,by the relation:

$$N_f = 8*\tan(\text{Theta}/2)/(\text{PI}*\text{Delta Theta})$$

The number cf fringes is important only to ensure that the signal processing apparatus will receive enough information to make possible further data reduction. If an insufficient number of fringes exists, not enough signal will be available for velocity determination.

Solid particles with a mean diameter which is less than half the fringe spacing are introduced into the free stream



FRINGE PATTERN GEOMETRY

Figure 1

of the fluid to be measured. When the particle is small ccmpared to the fringe spacing the scattered light exhibits essentially 100% modulation, while particles equal to the fringe spacing produce negligible modulation[Ref. 2]. The particles may be produced by atomization of water, dop-air reaction, burning oil, smoke bombs, or may be bought commercially, as e.g. polystyrene spheres and some solid chemicals such as zinc oxide. Choice of the particle to be used is normally dictated by the fringe spacing in the focal volume, and the nature of the medium to be measured. Regardless of choice, the particle must be small enough to follow the streamlines as closely as possible, because the laser anemometer actually measures the particle velocity and unless the particle's motion is a true representation of the fluid velccity, pertinent data may not be obtained.

Inside the measuring (or focal) volume, bands of darkness and brightness exist due to the interference cf the two beams. Particles entering the focal volume will alternately scatter light as they pass through regions of constructive interference (typical focal volume dimensions are 300 microns wide by 4 millemeters long and contain approximately 64 fringes). A photomultiplier tube located on the optical axis and focused into the measuring volume receives this scattered light and converts what appears to be a modulated light signal into a series of voltage pulses. Each voltage pulse correlates to a particle crossing one fringe. Because the spacing of the fringes is known, the frequency of the voltage pulse train may be equated to the velocity of the particle by the following relationship:

$$f = V_o / D_f = 2 * V_o * \sin(\text{Theta}/2) / \text{lambda}$$

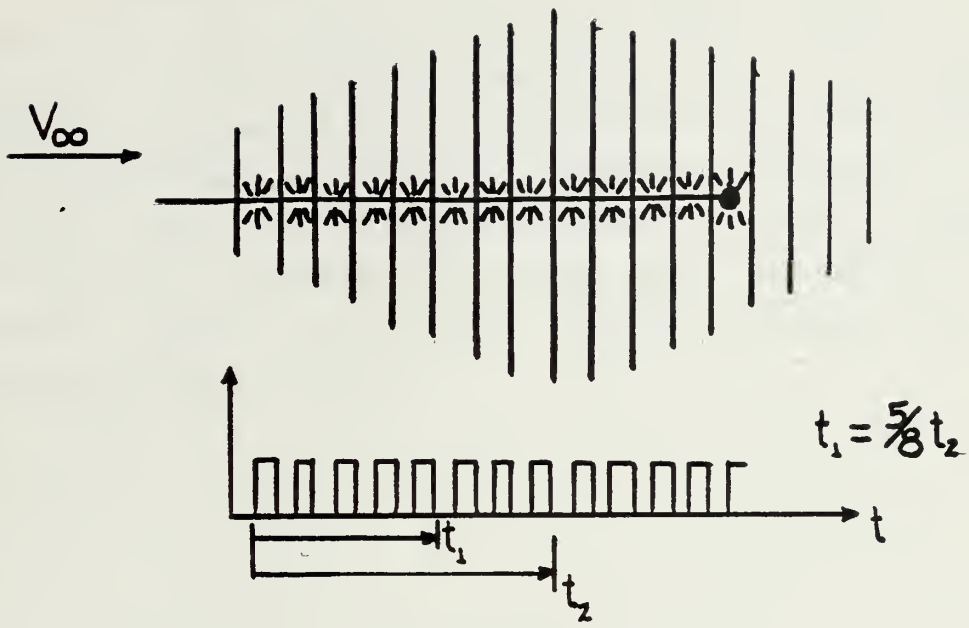
where D_f is the fringe spacing and V_0 is the velocity of the

particle. Now the processor is left with a fairly simple task of counting the frequency and multiplying it by a calibration factor, which is, in essence, a fringe spacing parameter to arrive at a velocity.

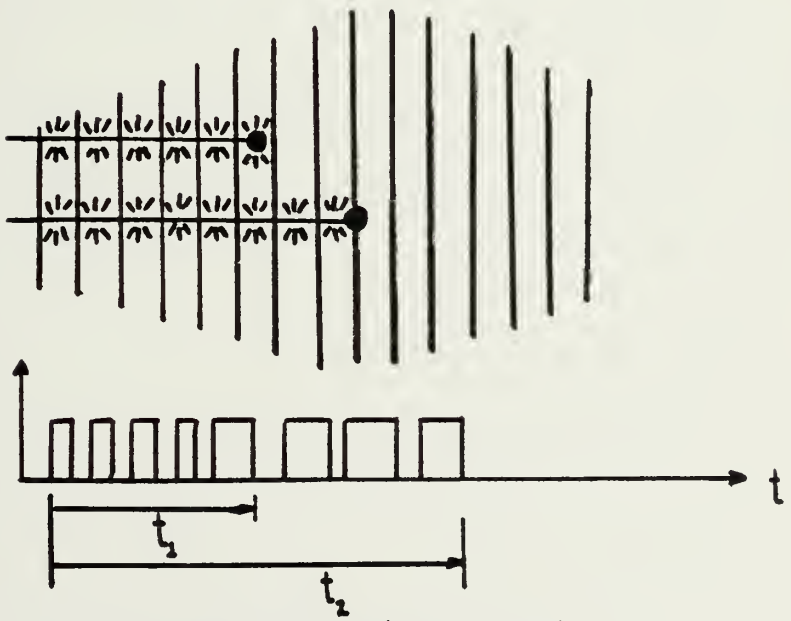
As long as we are assured that only one particle is in the measuring volume at one time, the apparatus will operate correctly, but this is an unrealistic assumption. In order to achieve such a situation, the seeding density of the particles would be exceptionally sparse and the resulting data rate would be so slow as to preclude timely data reduction. To overcome this obstacle, a validation circuit is incorporated into the counter/processor. When a particle crosses a fringe spacing, two counters are triggered. One measures the time duration for five fringe crossings and holds this information. The second counter continues to measure the time for a total of eight fringe crossings. The time of the first counter is then compared with that of the second counter, and if the first time is indeed five-eights of the second time (within specified limits, i.e., comp. accuracy) a validation gate is opened and the resultant frequency is then stored into an ensemble cell. The ensemble cell is adjustable to hold various numbers of counts and average them for the final velocity display. The final displayed velocity then is that component of fluid velocity which is perpendicular to the optical axis and contained in the same plane as the beam pair (Fig. 2).

The anemometer may be operated in three modes: reference beam, forward scatter, and back scatter. The last two modes are most frequently employed and are described here:

Forward scatter mode: in this mode, the photomultiplier tube is located on the optical axis and focused in a direction directly opposed to the direction of the emerging beam pairs. This mode offers the strongest attainable



validated signal

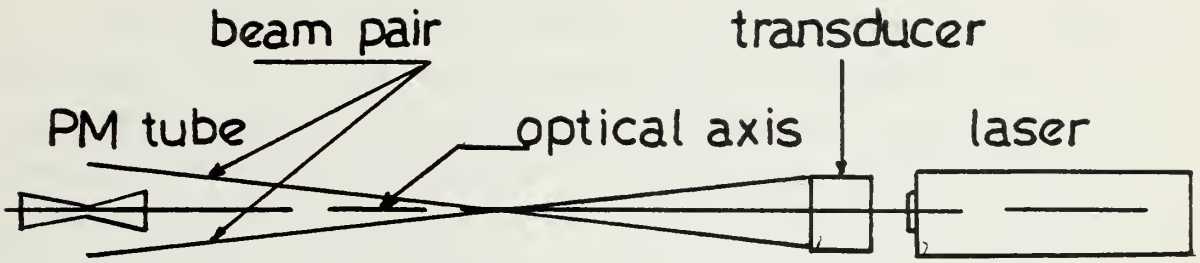


non-validated signal

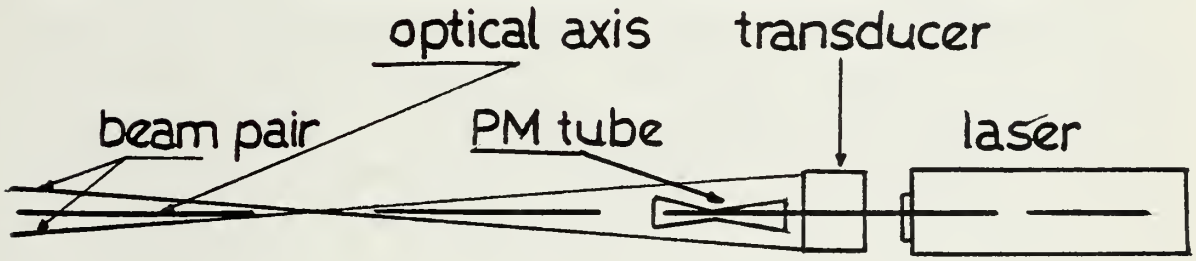
Figure 2

signal.

Back scatter mode: again the photomultiplier tube is mounted on the optical axis, but now it is aligned in the direction of the emerging beam pairs. This mode affords the ability tc measure fluid velocities in confined spaces or in situations where the dectector cannot be mounted in fcrward scatter, such as in measuring velocities in a compressor. Additionally, operation in this mode requires a higher powered laser due to the reduced intensity of back scattered light (Fig. 3).



LDV FORWARD SCATTER MODE



LDV BACK SCATTER MODE

Figure 3

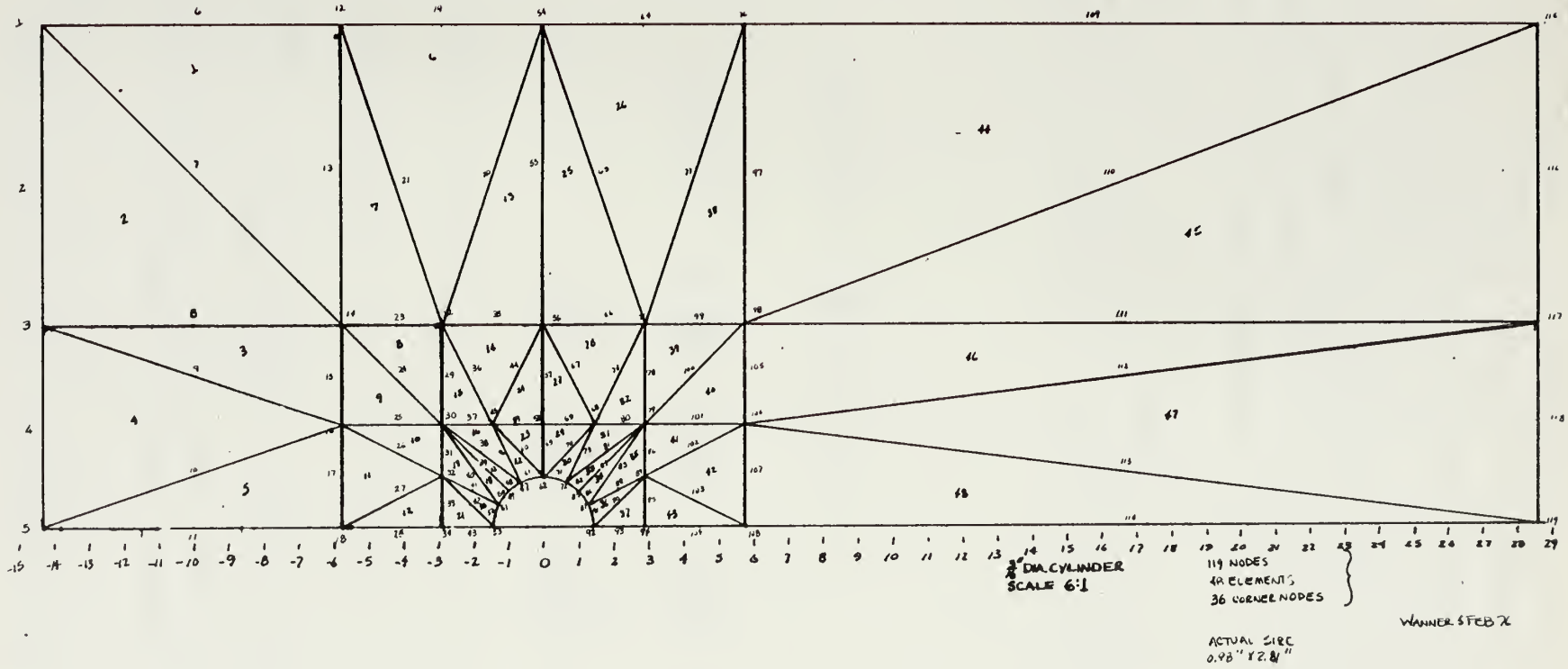
III ANALYTICAL DERIVATION

The finite element method is a numerical analysis technique fcr obtaining approximate solutions to a wide variety of enginering problems. It offers a means to solve complex continuum problems by sub-dividing them into simpler interrelated problems. The nature of this engineering problem does not readily lend itself to a closed form solution. Therefore a numerical solution is dictated.

The method of finite elements is well employed where there are irregular geometries or unusual specifications of boundary conditions. The method gives a piecewice approximation to the governing equations. Figure 4 shows the finite element grid employed in this flow problem. Approximating functions (interpolation functions) are defined in terms of the values of the field variables at the nodes. For the finite element representation, the nodal values of the field variables become the new unknowns and, once found, the interpolation functions define the field variable throughout the field of elements. The degree of the interpolatien polynomials is governed by the number of nodes per element and dictated by the order of the governing equations.

The basic principle of the finite element method is to divide the solution domain into a finite number of subdomains (elements) which are connected only at node points. Interpolation polynomials are created for each element and a collecton of interpolation functions over the whole solutien domain provide a piecewise approximation to the field variable.

Introduction of generalizations facilitate direct derivation of finite element equations from governing differential equations. The method of weighted residuals



Finite element grid

Figure 4

(Galerkin's Method) is a technique for obtaining approximate solutions to non-linear partial differential equations. We first assume the general function behavior of the dependent field variable in some way so as to approximately satisfy the given differential equations and boundary conditions. After applying this approximation to the original equations, some error will result, which we desire to minimize or drive to zero over the entire solution domain.

The basic set of differential equations governing the problem from a velocity and pressure formulation approach are (after linearization by approximating non-linear viscous terms u_n, v_n):

$$\text{continuity} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\text{momentum} \quad u_n \frac{\partial u}{\partial x} + v_n \frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu \nabla^2 u$$

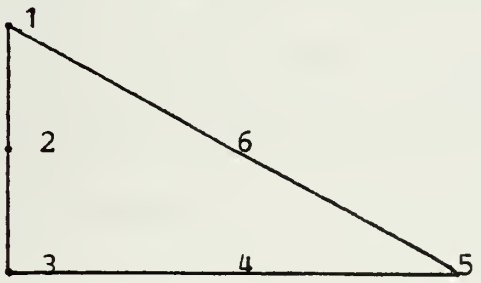
$$u_n \frac{\partial v}{\partial x} + v_n \frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu \nabla^2 v$$

u_n, v_n, and P_n are approximate solutions to the flow problem and may be visualized as the n^{th} iteration of the procedure of solution. These equations used in formulation of the problem are by no means unique. Stream functions and stream function with vorticity formulations have also been employed to establish finite element solution procedures.

Inasmuch as the velocity terms are of second order, a second order interpolation polynomial must be used to

calculate these unknowns. The pressure terms are of first order and hence require only first order interpolation.

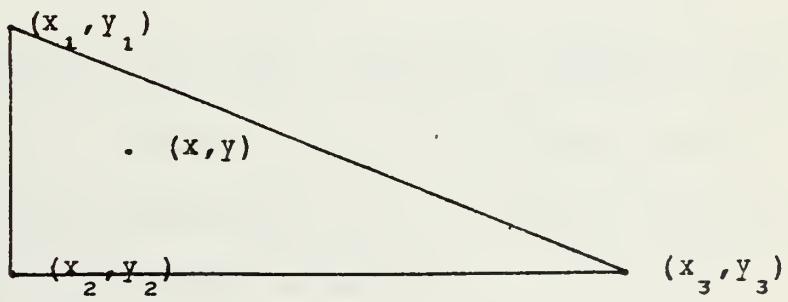
The basic element chosen for this solution was the triangle and, in a localized scheme, is visualized as follows:



Nodes 2, 4 and 6 lie equidistant between their respective corner nodes.

Nodes 1, 3, and 5 are referred to as corner nodes and are the basis for the linear interpolation polynomials derived for the pressure terms. For any triangular element, the corner nodes are described by their Cartesian coordinates x_i, y_i.

Consider the following local element:



The coordinates of node one are x_1, y_1; node two x_2, y_2; etc. and the interior point (x,y) may be described as a linear combination of the three corner nodes:

$$x = L_1 x_1 + L_2 x_2 + L_3 x_3$$

$$y = L_1 y_1 + L_2 y_2 + L_3 y_3$$

where L_i is the natural coordinate (interpolation polynomial, here linear) or weighting function which relates the coordinates of the corner nodes to any interior node. L_1 has a value of one at the point x_1, y_1 and is zero at the other two nodes of the element.

The L_i's expressed as linear functions of the Cartesian coordinates then are

$$L_1(x,y) = \frac{1}{2\Delta}(a_1 + b_1 x + c_1 y)$$

$$L_2(x,y) = \frac{1}{2\Delta}(a_2 + b_2 x + c_2 y)$$

$$L_3(x,y) = \frac{1}{2\Delta}(a_3 + b_3 x + c_3 y)$$

where $2\Delta = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$

which is equal to the area of the triangular element and $a_1 = x_2 y_3 - x_3 y_2$, $b_1 = y_2 - y_3$, and $c_1 = y_1 - y_2$, the determinants of the minors corresponding coefficients a_1, b_1, or c_1, and the other coefficients are similarly computed.

For the velocity terms, second order interpolation polynomials are indicated, while the pressure terms remain first order. Hence the introduction of more nodes (6) will result in a new set of weighting functions which may also be

expressed in terms of the natural coordinate interpolation polynomials:

$$N_1 = 2L_1^2 - L_1 \qquad N_2 = 4L_1L_2$$

$$N_3 = 2L_2^2 - L_2 \qquad N_4 = 4L_2L_3$$

$$N_5 = 2L_3^2 - L_3 \qquad N_6 = 4L_1L_3$$

$$N_i^P = L_i$$

$$u^e = \sum_1^6 N_i u_i \qquad v^e = \sum_1^6 N_i v_i \qquad P^e = \sum_1^3 N^P P_i \qquad \text{(xxxx)}$$

Application of Galerkin's Method to the governing equations results in the following intergral expressions:

$$-\int_\Omega N_i \left(u_n \frac{\partial u}{\partial x} + v_n \frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial x} - \nu\nabla^2 u\right) d\Omega = 0$$

$$-\int_\Omega N_i \left(u_n \frac{\partial v}{\partial x} + v_n \frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial P}{\partial y} - \nu\nabla^2 v\right) d\Omega = 0$$

$$\int_\Omega N_i^{(P)} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) d\Omega = 0$$

here Ω implies "over the area of the triangle".

Then we may make the substitution of the field variable in terms of x and y into terms of L_i.

e.g.: given $u(x,y) \Leftrightarrow u(L_i)$

then $$\frac{\partial u}{\partial x} = u^{(e)} \frac{\partial N_i}{\partial L_i}\frac{\partial L_i}{\partial x}$$

where $$\frac{\partial L_i}{\partial x} = \frac{b_i}{2\Delta}$$

and a similar procedure for $$\frac{\partial u}{\partial y} = u^{(e)} \frac{\partial N_i}{\partial L_i}\frac{\partial L_i}{\partial y}$$

where $\frac{\partial L_i}{\partial y} = \frac{c_i}{2\Delta}$

Substitution of equations (xxxx) into u^e, v^e and P^e and application of the above chain rule allows integration by parts:

$$K_1 = \int \left[\frac{\mu}{\rho} \left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} \right) - u^{(e)} N_i \frac{\partial N_j}{\partial x} - v^{(e)} N_i \frac{\partial N_j}{\partial y} \right] dx\, dy$$

$$K_2 = - \int N_j^{(r)} \frac{\partial N_i}{\partial x}\, dx\, dy$$

$$K_3 = - \int N_j^{(m)} \frac{\partial N_i}{\partial y}\, dx\, dy$$

The evaluated integral expressions then form the influence matrix as fcllows:

$$\begin{bmatrix} K_1 & 0 & K_2 \\ 0 & K_1 & K_3 \\ K_2 & K_3 & 0 \end{bmatrix}$$

Specification of the field variables or right hand sides of the matrix equation then provides a defined problem:

$$[T][T] = [Q]$$

Additionally, evaluation of the integral expressions yields boundary terms which must be computed for nodes which lie on the boundary of the control surface:

$$R_1 = \int N_i X^* ds \qquad R_2 = \int N_i Y^* ds$$

where $X^* = \mu \nabla u^{(e)} \cdot \hat{n} - \hat{n}_x P^{(e)}$ and $Y^* = \mu \nabla v^{(e)} \cdot \hat{n} - \hat{n}_y P^{(e)}$

It should be noted that along vertical boundaries there is

no outward pointing normal $\hat{n}_y$ and similarly along horizontal boundaries no $\hat{n}_x$ exists, and furthermore these terms are not computed where u and v are specified. Additionally, it was assumed that along the left-and right-hand side and upper boundaries $\partial u/\partial x$ and $\partial v/\partial y$ were identically equal to zero because these boundaries were placed at five or more characteristic lengths (diameters) from the cylinder and considered to be outside the realm of influence of the flow about the cylinder.

The terms comprising K_1, K_2 and K_3 are entered element by element into the main program in steps FLU02080 through QZ00170. Then each element's matrix is compiled into the field matrix (comprised of all elemental matricies) and is solved iteratively to output the horizontal velocity component u, the vertical component v, and the pressure P at every node in the field.

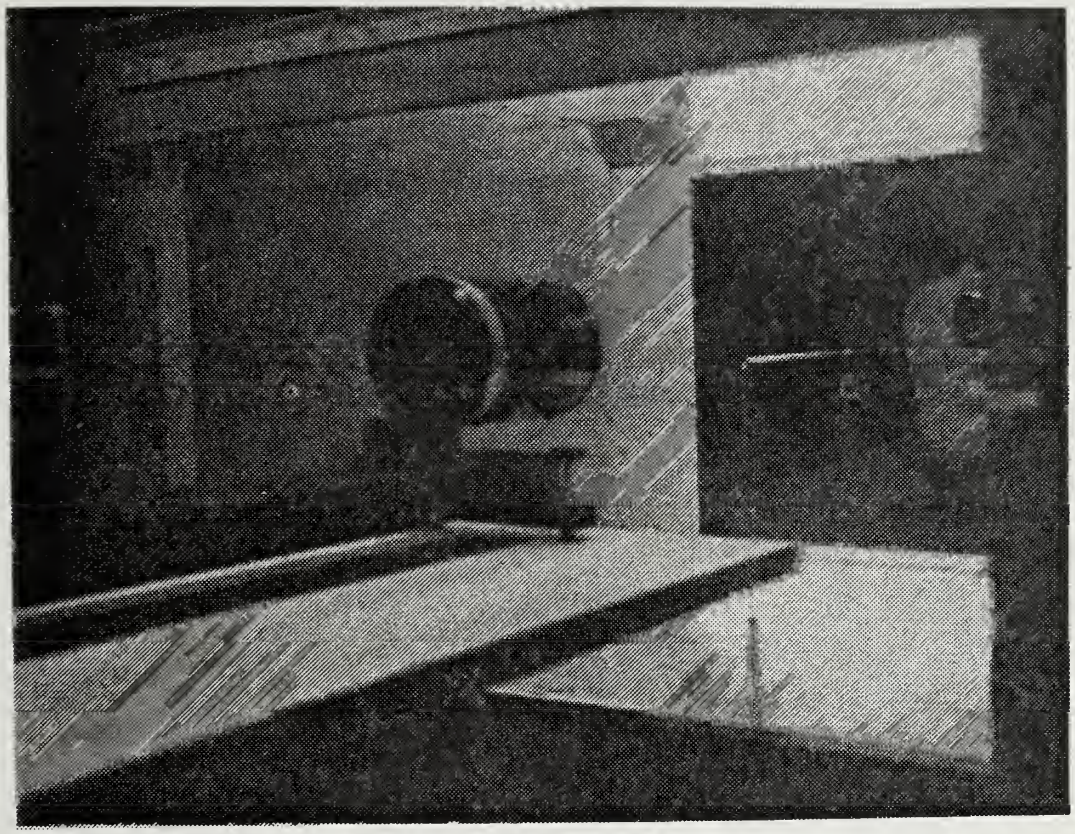
IV. DESCRIPTION OF EXPERIMENT

A right-angled, three-sixteenths inch cylinder made of stainless steel was mounted in an AEROLAB series 90 32x45 inch low speed wind tunnel through the floor of the test section (Fig. 5). The portion of the rod protruding down through the test section was mounted on a two-degree-of-freedom, micrometer controlled bed which allowed the rod to be maneuvered in the vertical plane parallel to the flow direction. This scheme enabled the laser velocimeter and photomultiplier tube to be mounted, aligned and left undisturbed for the entire period during which data were taken. The bed was positioned to read locations in the control volume which directly coincided with the location of the nodes in the finite element mesh (Fig. 6).

In the interest of precluding as many opportunities for error as possible, all u velocity components were first read and then components in a forty-five degree position were taken thus necessitating only one re-orientation of the beam pairs.

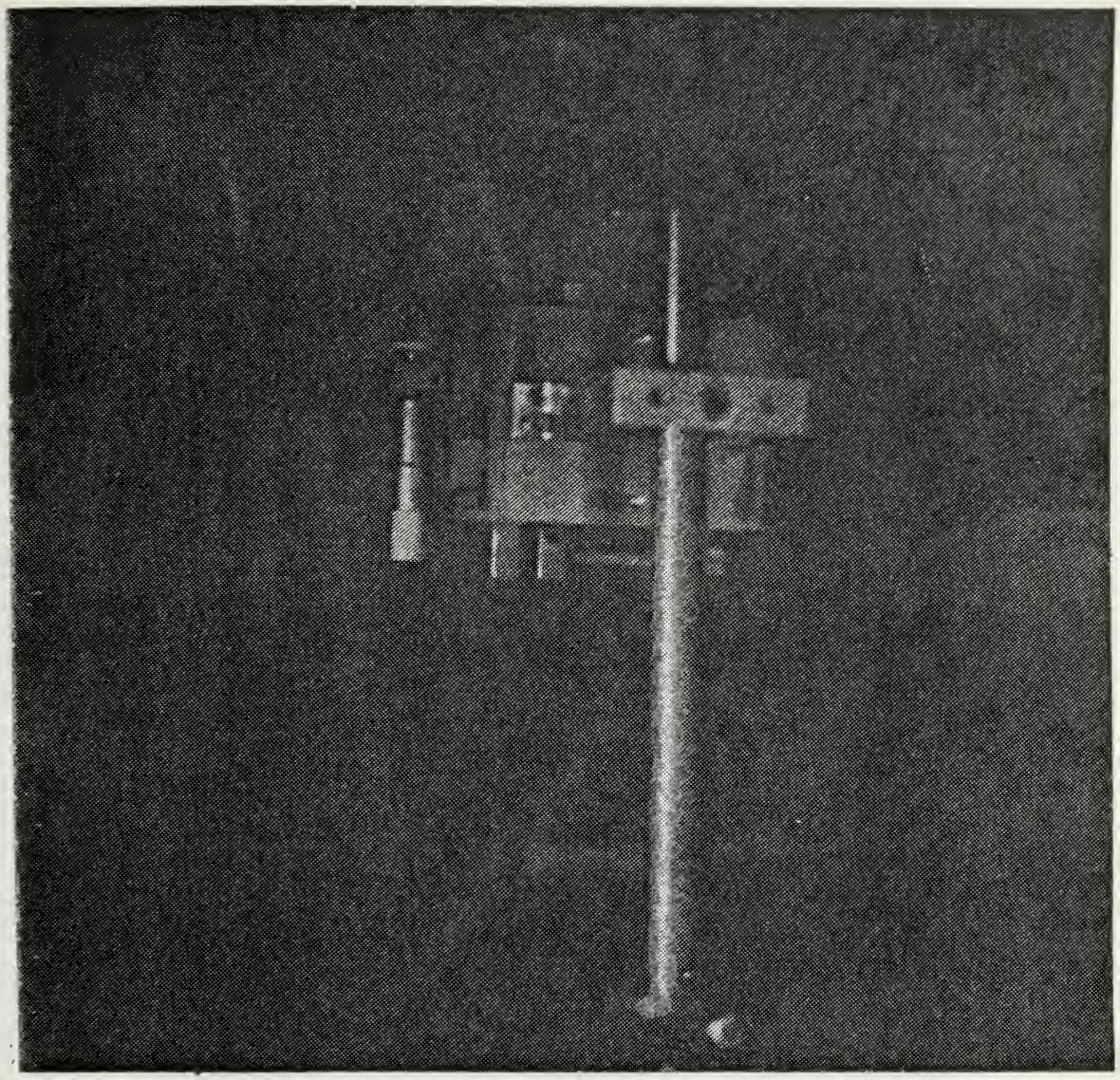
The plexiglass window prefacing the optics transducer of the test section had one-quarter inch holes drilled into it to allow the beam pairs to pass unimpeded, thereby minimizing distortion of the focal volume fringe pattern. The photomultiplier tube was likewise accommodated through the far window.

The seeding mechanism employed zinc-hexaclorcthane based (non-toxic) white smoke bombs. The nature of the closed return wind tunnel permitted continuous measurement without interruption for re-seeding of the flow. By means of electron microscopy, the average size of the smoke particles was determined to be on the order of two microns (Fig. 7).



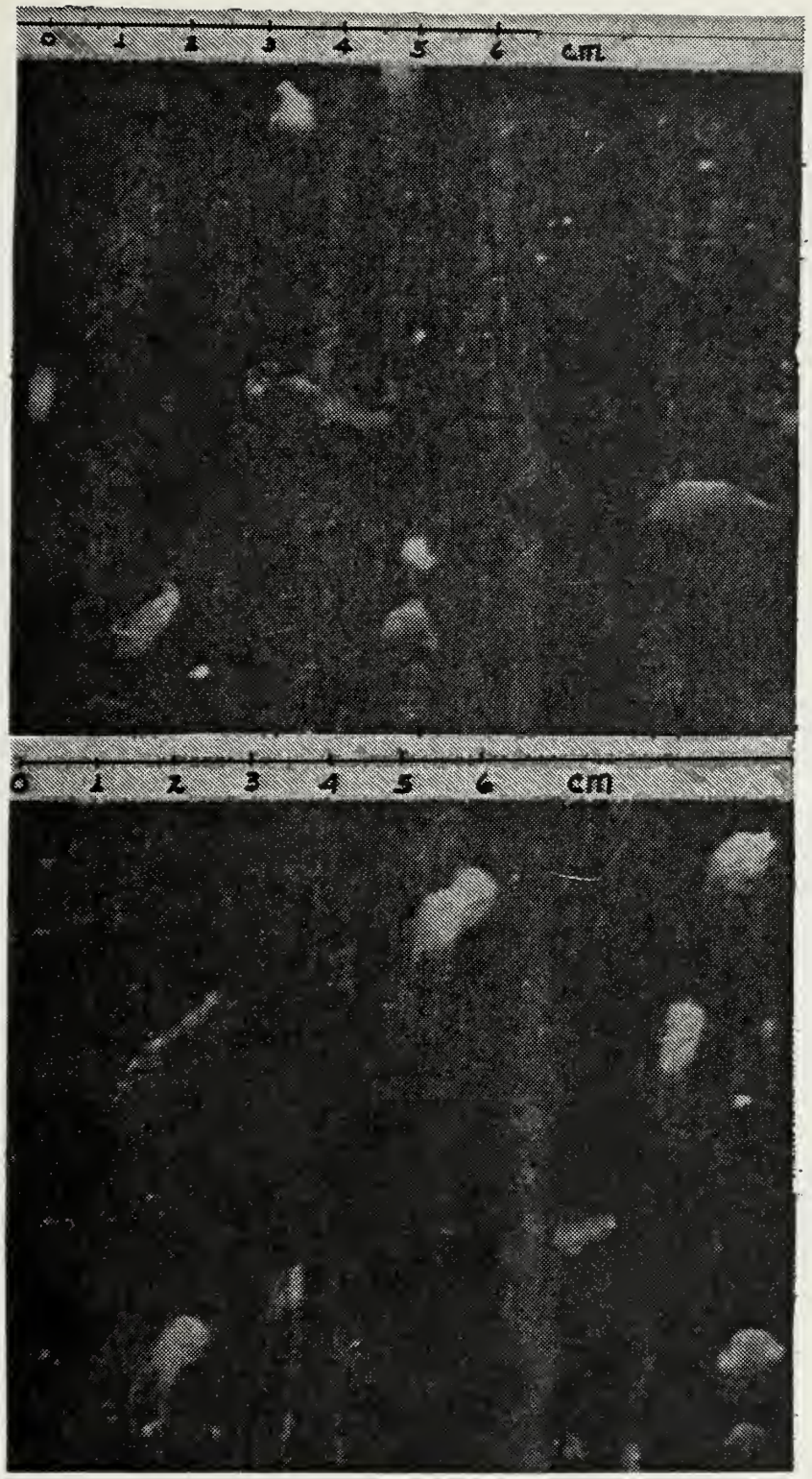
Cylinder mounted in wind tunnel and LDV setup.

Figure 5



Traversing mechanism

Figure 6

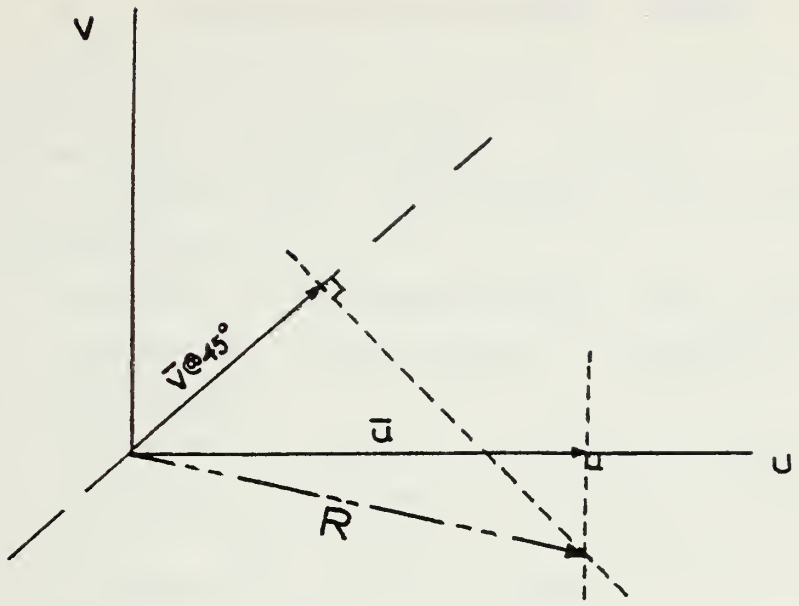


Zinc-hexachlorathane particles
(X5000 1cm= 2.0μm)

Figure 7

The final velocity component measured at each node was the result of averaging three sucessive readings from the velocimeter processor. Each of these readings represented the average of 4096 readings which had been validated to within 1.5% of the exact five-eighths validation count criteria.

The flow in the wind tunnel was stabilized at five inches per second to maintain a Reynolds number of about 40, which is in the realm of creeping flow. The experimental setup was designed to measure the steady flow about the cylinder, hence the relatively low Reynolds number, and no effort was made to obtain unsteady flow measurements. Three separate runs were conducted, two over the forward half of the control volume and one over the rear half. Because the beams of the laser anemometer were approximately one-sixteenth inch in diameter, several node position measurements were excluded due to inaccessibility. The majority of these were in close proximity to the cylinder. In order to obtain both an x- and a y-velocity component of the flow at every node which was accessible, two measurements were made, but because of the very small vertical component present in most of the flow, direct vertical beam pair measurement was unsuitable. Instead, after obtaining the horizontal component for each node in a run, a second series of measurements was made with the beam pairs inclined forty-five degrees from the vertical. In this fashion the y-component was determined by the following relationship:



It can be seen here that the unknown resultant (R) has a projection onto each axis representing the orientation of the beam pairs, thus the resultant is the vector which emanates from the origin and terminates at the intersection of the perpendiculars constructed from the $\bar{u}$ and $\bar{v}$ (at 45 degrees) vectors.

V. DESCRIPTION OF COMPUTER PROGRAM

The program found in the appendix has several comment cards inserted for clarification. However, a broader presentation is offered here. The program may be subdivided into four main sections:

Section 1: dimensioning and zeroing of parameters and arrays. (FLU00020 through FLU00690)

Section 2: input of node coordinates, system topology and known values of u, v, P, and right hand sides, and output of the same. (FLU00700 through FLU02090)

Section 3: computation of interpolation polynomials on an elemental level and arrangement into local element arrays. (FLU02100 through G000320)

Section 4: loading of the global influence matrix (TM) and solution of (TM)(T)=(Q) by Gauss-Jordan partial pivoting and back substitution, with an iterative routine incorporated until convergence is attained, and associated print out. (FLU04470 through FLU04880)

The computer routine systematically tested the solution of field variables with the previous solution until all terms in the T and T1 arrays agreed within 0.001 (ESPILA). If the convergence test failed, the process was repeated from section 3.

Initially, the program was run on an IBM 360 in FORTRAN CLG mode, then an ensuing run was completed on the FORTRAN HCLG compiler and a time reduction of two-thirds was realized. The program, as given in the appendix, required approximately 375K bytes of core to compile and 675K bytes

to execute and output. Substantial space savings may be realized by utilizing banded matrix storage methods vice the method employed here, but due to time limitations this avenue was not fully explored.

The program attempts to solve simultaneously for all the field variables. However, introduction of zeros on the main diagonal of the TM influence matrix resulted in a singular matrix. Another approach was explored where the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

was replaced by

$$-\frac{1}{2}\nabla^2 P = \left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} - \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} \right]$$

and the resulting terms were added to the TM matrix and their respective right hand sides. The only field variables affected by this form of the continuity equation were the pressure variables, and depending on whether the local corner node involved in the local pressure equation was on the boundary or interior, different right hand sides had to be evaluated. This is the purpose of steps FLU04460 through G000320. The simultaneous solution technique for all field variables applied to this new set of equations did not converge.

A third, unimplemented technique may be considered. Given the original matrix equations:

$$(1) \quad K_1 U + 0 + K_2 P = R_1$$

$$(2) \quad 0 + K_1 V + K_3 P = R_2$$

$$(3) \quad K_2 U + K_3 V + 0 = 0$$

Solution for u and v will result in

$$(1) \quad U = -K_1^{-1} K_2 P + K_1^{-1} R_1$$

$$(2) \qquad V = -K_1^{-1} K_3 P + K_1^{-1} R_2$$

and multiplication by K_2 and K_3 yields

$$K_2 U = -K_2 K_1^{-1} K_2 P + K_2 K_1^{-1} R_1$$

$$K_3 V = -K_3 K_1^{-1} K_3 P + K_3 K_1^{-1} R_2$$

Summation of these equations allows substitution for the third matrix equation:

$$\left[K_2 K_1^{-1} K_2 + K_3 K_1^{-1} K_3\right] P = K_2 K_1^{-1} R_1 + K_3 K_1^{-1} R_2$$

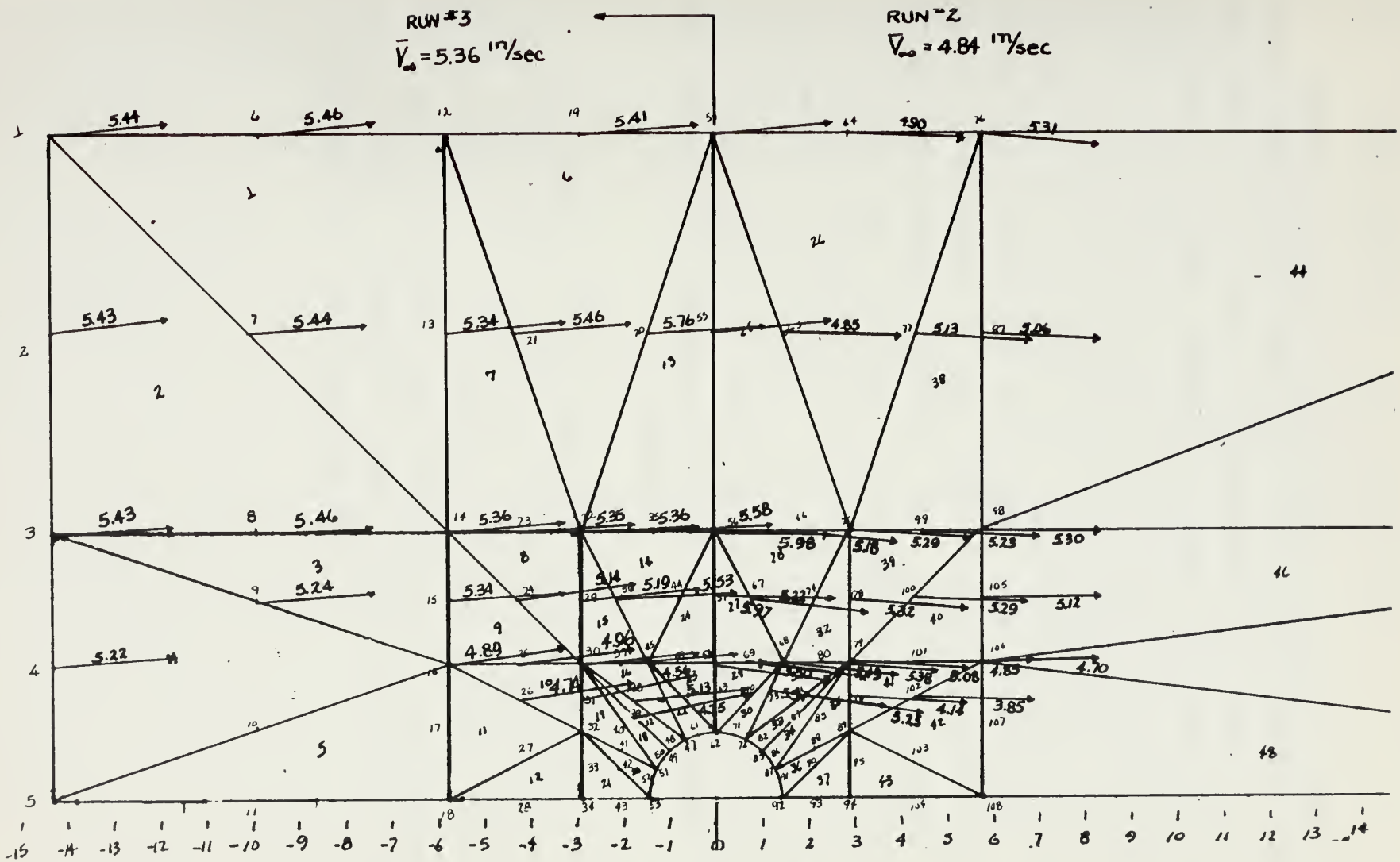
The matrix operating on P may be calculated on an algebraic level and entered, in terms of a_i, b_i and c_i, into the program in order to solve for the pressures at every corner node. The calculated P's then would be entered into the field variable array and the partial pivoting routine, which involves only those terms where the field variable is not specified, would operate primarily on the first two matrix equations. In previous computer trials where all corner node pressures were specified, although not known to be correct, calculations were properly performed. This method would allow the actual pressures present in the control surface to be entered into the computer routine for further calculation of the velocity components.

VI. ANALYSIS OF RESULTS

No computer routine output was obtained for comparison of results, due to the inability of the program to converge. As such, comparison of the two methods cannot be offered here. The non-convergence of the program was investigated from several aspects: validity of the non-linear terms, correctness of the loading of the TM matrix on both the elemental and global levels, correctness of input, performance of the partial pivoting subroutines and the iteration scheme; all without success. In test cases the program initially solved for the unknowns within magnitude accuracy for a majority of terms. However, sucessive iterations resulted in oscillating solutions, even after twenty iterations. Convergence was expected after ten to fifteen iterations without under- or overrelaxation factors incorporated into the iteration scheme. After the new formulation of the continuity equation was incorporated into the computer routine, the program execution again resulted in a singular matrix when only the left and right hand and upper control surface boundaries had their respective pressures specified. Underflow occurred in the execution of the subroutine SOLVE at step SOL00260. Subsequent investigation revealed that the 178^{th} and 179^{th} rows of the reduced matrix equations were the cause of the underflow.

The data which were taken for runs number two and three are displayed in the appendix and are plotted on the finite element grid in Figure 8. Without the analytical results to compare the measured data only a qualitative statement may be made pertaining to the LDV output. It is readily seen that the trend of flow directions around the cylinder appears to be correct. By and large, the flow in close proximity to the cylinder was accelerated over the top and decelerated near the stagnation points. It is pointed out

here that the plot is a composite of two different trials, and as such, different free stream velocities were present for each run. In Figure 8 the vector representing the flow at a node was plotted and the magnitude of the vector in inches per second was displayed above the vector or by the node from which the velocity vector emanated.



Velocity data plot

Figure 8

A. ERROR ANALYSIS

Re-positioning of the cylinder to its base location was accomplished with relatively low position error. Prior to each run the cylinder was aligned using the beam pairs as reference. In the horizontal mode, the beam pairs would strike the leading and trailing portions of the cylinder and reflect to positions horizontally opposed on the far side of the wind tunnel when correctly zeroed vertically. The same procedure was repeated for the vertical beam pair except position was checked by the upper beam reflecting to a vertical position on the far wall and the lower beam (at reduced intensity) striking the cylinder support on centerline. This procedure was repeated several times and the maximum deviation between any aligned positions was 0.2 millimeters horizontally or vertically. Thus, it was determined that the maximum positioning error inherent in the system would be the ratio of the smallest coordinate of any node measured and the aforementioned 0.2 millimeters or

$$0.2/3.75=0.5\%$$

Freestream velocity measurements were likewise tested for fluctuations. Typical measurement variations ranged from 0.129 to 0.142 (run number three; freestream measurements) in the horizontal mode and 0.0947 to 0.100 in the forty- five degree mode (velocities in meters/seconds). These figures represent a total of 10% variation which may be considered as ± 5.% from the mean.

VII. CONCLUSION

The experiment, as described, required approximately two to four hours for each run. aside from the initial alignment of the components and the cylinder, this was perhaps the greatest drawback of the system. Next, the exclusion of regions in close proximity to the cylinder from measurement was deemed to be a major factor in the small amount of data actually taken. The latter may be improved with a different experimental arrangement; particularly, the plexiglass section directly prefacing the optics transducer should be completely removed, thus allowing all angles of beam pair inclination, and in turn availing more locations to measurement. Another improvement which could be implemented would be unitization of the laser, transducer and photomultiplier tube arrangement. This would decrease the LDV's sensitivity to vibration and provide a more accurate optical axis alignment. The aforementioned improvements would still not decrease the time required for data collection, but the introduction of a two component system [Ref.4] would allow simultaneous measurement using orthogonal beam pairs. Furthermore, this type of system would not concern itself with the errors introduced in beam pair re-alignments performed with the single beam pair LDV.

The reproducibility of the experiment was judged to be good with the exception of exact duplication of the wind tunnel velocity between successive trials. This, of course, is primarily a function of the controllability of the tunnel motor and propellor assembly. It is estimated that with adequate facilities for incorporation of some of the aforementioned improvements, more accurate data may be obtained. The LDV displays great potential as a means of measuring a wide variety of dynamic fluid regimes accurately and without introduction of disturbance to the flow itself.

APPENDIX A

LDV DATA

RUN #2

Node	U(cm/sec)	V@45°(cm/sec)	R(in/sec)	∠°
64	12.43	8.43	4.90	-2.3
65	12.33	8.49	4.85	-1.5
66	13.10	8.33	5.18	-5.8
67	13.17	8.25	5.22	-6.5
68	13.87	8.72	5.49	-6.3
69	13.87	8.56	5.50	-7.2
73	14.03	8.58	5.57	-7.7
75	13.40	8.83	5.29	-3.9
76	13.43	8.62	5.31	-5.3
77	13.00	8.62	5.13	-3.6
78	13.47	8.87	5.32	-4.5
79	12.87	8.58	5.08	-3.3
80	13.63	8.91	5.38	-4.3
81	13.20	8.02	5.25	-8.0
96	10.50	6.94	4.14	-3.7
97	12.83	8.64	5.06	-2.7
98	13.47	9.54	5.30	0.1
99	13.27	8.92	5.23	-2.8
100	13.43	9.25	5.29	-1.5
101	12.33	8.83	4.85	0.7
102	9.77	6.69	3.85	-1.8
105	13.03	9.32	5.12	0.8
106	11.93	8.68	4.70	1.7

RUN #3

Node	U(cm/sec)	V@45°(cm/sec)	R(in/sec)	°
1	13.73	10.77	5.44	6.23
2	13.70	10.43	5.43	6.25
3	13.77	10.33	5.43	3.49
4	13.23	10.03	5.22	4.11
6	13.77	10.93	5.46	7.00
7	13.80	10.40	5.44	3.77
8	13.87	10.17	5.46	2.11
9	13.27	10.08	5.24	4.27
13	13.50	10.47	5.34	5.54
14	13.57	10.40	5.36	4.80
15	13.53	10.20	5.34	3.76
16	12.30	9.93	4.89	8.05
19	13.70	10.43	5.41	4.38
20	14.60	10.87	5.76	3.02
21	13.83	10.57	5.46	4.63
22	13.60	10.06	5.36	2.65
23	13.57	10.23	5.35	3.79
24	12.93	10.40	5.14	7.84
25	12.50	9.92	4.96	7.00
26	11.90	9.64	4.74	8.27
29	13.13	10.16	5.19	5.40
30	12.57	9.80	4.98	5.86
31	11.30	9.76	4.56	12.50
35	14.17	10.47	5.58	2.60
36	14.03	10.43	5.53	4.36
37	13.23	10.07	5.22	4.36
38	12.97	10.09	5.13	5.72
39	11.83	10.03	4.75	11.24
54	14.03	10.87	5.55	5.58
55	14.17	11.03	5.61	5.76
56	15.20	10.57	5.98	-0.90
57	15.17	10.47	5.97	-1.36
58	16.27	10.00	6.46	-7.46
59	14.37	10.20	5.66	0.20

```
C
C
C
C   THIS RUN HAS KINEMATIC VISCOSITY =.022948 (INCHES**2)
C    THIS RUN (#2) HAS A REYNOLDS NO.= 39.59
C
        DATA NREAD/5/
C                                                                          FLU00020
C   THE U VELOCITY IS IN THE FIRST NN POSITIONS OF  T(I) (NN=NUMBER OFNODFLU00030
C   THE V VELOCITY IS IN THE SECOND NN POSITIONS OF T I.E. T(I+NN)         FLU00040
C   THE P PRESSURE IS IN THE  NN+NN+I POSITIONS OF  T  I.E.  T(I+NN+NN)   FLU00050
C     THERE ARE  NNCN  PRESSURE NODES (NNCN=NUMBER OF CORNER  NODES)       FLU00060
C                                                                          FLU00070
C    BOTH TM AND B MUST TO DIMENSIONED 2*NN+NNCN X 2*NN+NNCN               FLU00080
CC                                                                         FLU00090
C                                                                          FLU00100
          DATA NWRITE/6/                                                   FLU00110
        DATA STOP/'STOP'/                                                  FLU00120
C     PRESSURE READ IN FOR RUN # 2
        DATA PRES/14.8/
        DIMENSION TM(274,274),B(274,274)
        DIMENSION XC(120),YC(120),NODE(120,6),NVS(120),NCN(120)
        DIMENSION T(274),NVIS(274),NCP(120),NPS(120),Q(274)
        DIMENSION NQS(120),T1(274)
        DIMENSION TM$(15,15),N(15),NQIS(274)
        DIMENSION Z(274),C(274)
        DIMENSION RHS(274)
        DIMENSION RP$(6),ZP$(6)                                            FLU00200
        DIMENSION XC$(3),YC$(3)                                            FLU00210
          DIMENSION U(6),V(6),QZ1(6,6),QZ2(6,6),QZ3(6,6)                   QZ 00020
         DIMENSION NI(18),NT(3)
        DIMENSION NLCY(4),NRCY(4),QZB(6),QZC(6)                            GO 00010
C                                                                          FLU00220
C    IF NCASE EQUALS 1, THEN THE PROBLEM WILL INCLUDE
C      NCN-LINEAR TERMS AND A TWO DIMENSIONAL SOLUTION
C(                                                                         FLU00250
        READ(NREAD,500)NCASE                                               FLU00260
        IF(NCASE.EQ.1)GO TO 5                                              FLU00270
        GO TO 6                                                            FLU00290
     5  WRITE(NWRITE,2020)                                                 FLU00300
     6  CONTINUE                                                           FLU00310
C                                                                          FLU00320
C   THE FIRST PART OF THE PROGRAM CAN BE CONSIDERED AS AN INPUT ROUTINE    FLU00330
C   LINE 022 TO  0170 ARE INPUT AND VERIFICATION OF ALL DATA               FLU00340
C                                                                          FLU00350
C   THIS WOULD BE PART OF ANY FINITE ELEMENT PROGRAM                       FLU00360
CC                                                                         FLU00370
C                                                                          FLU00380
```

```
C    READ IN NUMBER OF NODES AND ELEMENTS AND NO. CORNER NODES          FLU00390
       READ(NREAD,1005)NN,NE,NNCN                                       FLU00400
C                                                                       FLU00410
C      INITIALIZE ALL PARAMETERS                                        FLU00420
C                                                                       FLU00430
         MM=NN+NN+NNCN                                                  FLU00440
       DO 50 I=1,120                                                    FIX00020
       XC(I)=0.                                                         FIX00030
       YC(I)=0.                                                         FIX00040
       NVS(I)=0                                                         FIX00050
       NCN(I)=0                                                         FIX00070
       NCP(I)=0                                                         FIX00080
       NPS(I)=0                                                         FIX00200
       NCS(I)=0                                                         FIX00090
    50 CONTINUE                                                         FIX00100
       DO 51 I=1,MM                                                     FIX00060
       NVIS(I)=0                                                        FIX00210
       NQIS(I)=0                                                        FIX00110
       T(I)=0.                                                          FIX00120
       Q(I)=0.                                                          FIX00130
       T1(I)=0.                                                         FIX00140
       Z(I)=0.                                                          FIX00150
       C(I)=0.                                                          FIX00160
       DO 51 J=1,MM                                                     FIX00170
       B(I,J)=0.                                                        FIX00180
    51 CONTINUE                                                         FIX00230
       DO 53 I=1,15                                                     FIX00240
       N(I)=0                                                           FIX00250
       DO 53 J=1,15                                                     FIX00260
       TM$(I,J)=0.                                                      FIX00270
    53 CONTINUE                                                         FIX00290
       DO 54 I=1,6                                                      FIX00300
       RP$(I)=0.
       ZP$(I)=0.
       U(I)=0.0
       V(I)=0.0
       DO 54 K=1,6
       QZ1(I,K)=0.0
       QZB(I)=0.0
       QZC(I)=0.0
       QZ2(I,K)=0.0
       QZ3(I,K)=0.0
    54 CONTINUE                                                         FIX00310
       DO 55 I=1,3                                                      FIX00320
       XC$(I)=0.                                                        FIX00330
       YC$(I)=0.                                                        FIX00340
       NT(I)=0                                                          FIX00350
    55 CONTINUE
```

```
      DO 56 I=1,4                                                        FLU00690
      NLCY(I)=0.0                                                        FLU00700
      NRCY(I)=0.0                                                        FLU00710
   56 CONTINUE                                                           FLU00720
C    READ NODE NUMBERS AND COORDINATES                                  FLU00730
C                                                                       FLU00740
      DO 100 J=1,NN                                                      FLU00750
      READ(NREAD,1006)WORD,I,XC(I),YC(I)                                 FLU00760
       IF (WORD.EQ.STOP) GO TO 101                                       FLU00770
        NCN(J)=I                                                        FLU00780
  100 CONTINUE                                                          FLU00790
  101   NNCN=J-1                                                        FLU00800
C                                                                       FLU00810
C   THE ARRAY  NCP(J) GENERATES THE GLOBAL PRESSURE INDICES (P1,P2.)    FLU00820
C   THUS PRESSURE NODES ARE LABELED AS CORNER NODES ARE INPUTTED        FLU00830
C      WHEN ONE INPUTS A GLOBAL CORNER NODE FOR J                       FLU00840
C                                                                       FLU00850
      DO 107 J=1,NNCN                                                   FLU00860
      NCP(NCN(J))        =J+NN+NN                                       FLU00870
  107 CONTINUE                                                          FLU00880
C                                                                       FLU00890
C    SYSTEM TOPOLOGY( ELEMENT NO. AND NODE NUMBERS IN                   FLU00900
C COUNTER-CLOCKWISE FASHION STARTING AT ANY CORNER NODE                 FLU00910
C ALWAYS COUNT FROM UPPER LEFT HAND CORNER                              FLU00920
C                                                                       FLU00930
      DO 105 I=1,NE                                                     FLU00940
      READ(NREAD,1010)J,NODE(J,1),NODE(J,2),NODE(J,3),                  FLU00950
     1NODE(J,4),NODE(J,5),NODE(J,6)                                     FLU00960
  105 CONTINUE                                                          FLU00970
         MAXDIF=0                                                       FLU00980
      DO 108 I=1,NE                                                     FLU00990
      DO 108 J=1,6                                                      FLU01000
      DO 108 K=1,6                                                      FLU01010
         LL=IABS(NODE(I,J)-NODE(I,K))                                   FLU01020
         IF(LL.GT.MAXDIF) MAXDIF=LL                                     FLU01030
      IBAND=2*(MAXDIF+1)                                                FLU01040
      NEQ=2*NN+NNCN                                                     FLU01050
  108  CONTINUE                                                         FLU01060
      WRITE(NWRITE,1017)IBAND,NEQ                                       FLU01070
 1017 FORMAT(5X,'IBAND=',I3,//,5X,'NEQ =',I3,//)                        FLU01080
C                                                                       FLU01090
C   READ NODES WHERE BOTH U AND V VELOCITY IS SPECIFIED                 FLU01100
C                                                                       FLU01110
      DO 110 I=1,MM                                                     FLU01120
      READ(NREAD,1006)WORD,NVELS,VELU,VELV                              FLU01130
       IF(WORD.EQ.STOP) GO TO 111                                       FLU01140
      NVS(I)=NVELS                                                      FLU01150
      T(NVS(I))=VELU                                                    FLU01160
```

```
      T(NVS(I)+NN)= VELV                                                    FLU01130
  110     CONTINUE                                                          FLU01140
C                                                                           FLU01150
CC   COUNT NODES HAVING SPECIFIED  VELOCITIES                               FLU01160
C                                                                           FLU01170
  111 NNVELS= I-1                                                           FLU01180
C                                                                           FLU01190
CC                                                                          FLU01200
CC                                                                          FLU01210
C READ QX AND QY VALUES AT INTERNAL NODES                                   FLU01220
      DO 125 I=1,NN                                                         FLU01230
        READ(NREAD,1006)WORD,NQXY,QXNS,QYNS                                 FLU01240
        IF(WORD.EQ.STOP) GO TO 126                                          FLU01250
        NQS(I)= NQXY                                                        FLU01260
        Q(NQS(I))=QXNS*PRES
        Q(NQS(I)+NN)=QYNS*PRES
  125 CONTINUE                                                              FLU01290
  126   NNQXY=I-1                                                           FLU01300
C                                                                           FLU01310
CC   READ NODE NUMBERS AND  PRESSURE WHEREE PRESSURE  IS SPECIFIED          FLU01320
C                                                                           FLU01330
      DO 130 I=1,NN                                                         FLU01340
      READ(NREAD,1025)WORD,NP,PNP                                           FLU01350
      IF(WORD.EQ.STOP)GO TO 135                                             FLU01360
      NPS(I)= NP                                                            FLU01370
      T(NCP(NPS(I)))= PNP                                                   FLU01380
  130 CONTINUE                                                              FLU01390
C                                                                           FLU01400
CC COUNT  BOUNDARY NODES WHERE PRESSURE SPECIFIED                           FLU01410
C                                                                           FLU01420
  135 NNPS= I-1                                                             FLU01430
C                                                                           PRE00100
CC                                                                          PRE00090
C READ PRESSURE NODES WHERE Q IS SPECIFIED                                  PRE00110
CC                                                                          PRE00130
C                                                                           PRE00010
      DO 141 I=1,MM                                                         PRE00020
      READ(NREAD,1025)WORD,NPQ,QNPQ                                         PRE00030
      IF(WORD.EQ.STOP)GO TO 142                                             PRE00040
      NPS(I+NNPS)=NPQ                                                       PRE00050
      Q(NCP(NPS(I+NNPS)))=QNPQ                                              PRE00070
  141 CONTINUE                                                              PRE00080
  142 NNPQ=I-1                                                              QZ 00160
C                                                                           QZ 00140
CC    READ IN CORNER NODES WHICH ARE CONTAINED IN THE INTERIOR              QZ 00070
C                                                                           QZ 00080
        DO 143 I=1,MM
        READ(NREAD,7654) NINT
```

```
        IF(WORD.EQ.STOP) GO TO 147                                          QZ 00100
        NI(I)=NINT                                                          8Z 00 10
  143 CONTINUE
 7654 FORMAT(6X,A4,14I4)
  147 CONTINUE
C                                                                           FLU01440
C NQIS IS A LIST OF THE INDICES OF KNOWN QX QY                              FLU01450
C                                                                           FLU01460
        DO 1140 I=1,NNQXY                                                   FLU01470
        NQIS(I)=NQS(I)                                                      FLU01480
         NQIS(I+NNQXY)=NQS(I)+NN                                            FLU01490
 1140  CONTINUE                                                             FLU01500
      DO 1142 I=1,NNPQ
      NQIS(I+2*NNQXY)=NPS(I+NNPS)+2*NN
 1142 CONTINUE
C                                                                           FLU01510
C  NVIS IS A LIST OF INDICIES OF KNOWN VELOCITIES AND PRESSURES
C                                                                           FLU01530
        DO 1150 I=1,NNVELS                                                  FLU01540
       NVIS(I)=NVS(I)                                                       FLU01550
       NVIS(I+NNVELS)=NVS(I)+NN                                             FLU01560
 1150 CONTINUE                                                              FLU01570
        DO 1160 J=1,NNPS                                                    FLU01580
      NVIS(2*NNVELS+J)=NCP(NPS(J))                                          FLU01590
 1160   CONTINUE                                                            FLU01600
C                                                                           FLU01610
C NTOTQ TOTAL NUMBER OF KNOWN QX AND QY                                     FLU01620
C                                                                           FLU01630
        NTOTQ=2*NNQXY +NNPQ                                                 FLU01640
        NTOTVP=2*NNVELS+NNPS
C                                                                           FLU01710
C  PRINT ALL INPUT DATA                                                     FLU01720
C                                                                           FLU01730
      WRITE(NWRITE,1035)NN,NE,NNCN                                          FLU01740
      WRITE(NWRITE,1040)                                                    FLU01750
      DO 150 I=1,NNCN                                                       FLU01760
       WRITE(NWRITE,1045)NCN(I),XC(NCN(I)),YC(NCN(I))                       FLU01770
  150   CONTINUE                                                            FLU01780
      WRITE(NWRITE,1050)                                                    FLU01790
      DO 155 I=1,NE                                                         FLU01800
      WRITE(NWRITE,1055)I,NODE(I,1),NODE(I,2),NODE(I,3),                    FLU01810
     1NODE(I,4),NODE(I,5),NODE(I,6)                                         FLU01820
  155 CONTINUE                                                              FLU01830
      WRITE(NWRITE,1060)                                                    FLU01840
      DO 160 I=1,NNVELS                                                     FLU01850
      WRITE(NWRITE,1065)I,NVS(I),T(NVS(I)),T(NVS(I)+NN)                     FLU01860
  160 CONTINUE                                                              FLU01870
      WRITE(NWRITE,1070)                                                    FLU01880
```

```
      DO 165 I=1,NNQXY                                                  FLU01890
       WRITE(NWRITE,1065)I,NQS(I),Q(NQS(I)),Q(NQS(I)+NN)                FLU01900
  165 CONTINUE                                                          FLU01910
      IF(NNPQ.EQ.0)GO TO 163
      WRITE(NWRITE,1071)                                                PRE00160
      DO 163 I=1,NNPQ                                                   PRE00170
      WRITE(NWRITE,1072)I,NPS(I+NNPS),Q(NCP(NPS(I+NNPS)))               PRE00180
  163 CONTINUE                                                          PRE00190
      WRITE(NWRITE,1080)                                                FLU01920
      DO 170 I=1,NNPS                                                   FLU01930
      WRITE(NWRITE,1085)I,NPS(I),T(NCP(NPS(I)))                         FLU01940
  170 CONTINUE                                                          FLU01950
  177 CONTINUE
       DO 179 I=1,MM                                                    FLU02030
        RHS(I)=0.0                                                      FLU02040
      DO 179 J=1,MM                                                     FLU02050
       TM(I,J)=0.                                                       FLU02060
  179    CONTINUE                                                       FLU02070
C                                                                       FLU02080
CC   END OF  INPUT AND VERIFICATION ROUTINE                             FLU02090
C                                                                       FLU02100
      DO 300 K=1,NE                                                     FLU02110
      N1=NODE(K,1)                                                      FLU02120
      N2=NODE(K,2)                                                      FLU02130
      N3=NODE(K,3)                                                      FLU02140
       N4=NODE(K,4)                                                     FLU02150
       N5=NODE(K,5)                                                     FLU02160
       N6=NODE(K,6)                                                     FLU02170
      N7 = NODE(K,1)+NN                                                 FLU02180
      N8 =NODE(K,2)+NN                                                  FLU02190
      N9 =NODE(K,3)+NN                                                  FLU02200
      N10=NODE(K,4)+NN                                                  FLU02210
      N11=NODE(K,5)+NN                                                  FLU02220
      N12=NODE(K,6)+NN                                                  FLU02230
      N13=NCP(NODE(K,1))                                                FLU02240
      N14=NCP(NODE(K,3))                                                FLU02250
      N15=NCP(NODE(K,5))                                                FLU02260
      XC$(1)=XC(NODE(K,1))                                              FLU02270
      XC$(2)=XC(NODE(K,3))                                              FLU02280
      XC$(3)=XC(NODE(K,5))                                              FLU02290
      YC$(1)=YC(NODE(K,1))                                              FLU02300
      YC$(2)=YC(NODE(K,3))                                              FLU02310
      YC$(3)=YC(NODE(K,5))                                              FLU02320
      A=.0229248
      AA=1.                                                             FLU02340
      A1=XC$(2)*YC$(3)-XC$(3)*YC$(2)                                    FLU02350
      A2=XC$(3)*YC$(1)-XC$(1)*YC$(3)                                    FLU02360
      A3=XC$(1)*YC$(2)-XC$(2)*YC$(1)                                    FLU02370
```

```
      B1=YC$(2)-YC$(3)                                                          FLU02380
      B2=YC$(3)-YC$(1)                                                          FLU02390
      B3=YC$(1)-YC$(2)                                                          FLU02400
      C1=XC$(3)-XC$(2)                                                          FLU02410
      C2=XC$(1)-XC$(3)                                                          FLU02420
      C3=XC$(2)-XC$(1)                                                          FLU02430
      DEL=ABS(0.5*(XC$(1)*(YC$(2)-YC$(3))+XC$(2)*(YC$(3)-YC$(1))+XC$(           FLU02440
     1   3)*(YC$(1)-YC$(2))))                                                   FLU02450
      CONST=(1.*A/(3.*DEL))*AA                                                  FLU02460
      D1=-B1/6.                                                                 FLU02470
      D2=-B2/6.                                                                 FLU02480
      D3=-B3/6.                                                                 FLU02490
      E1=-C1/6.                                                                 FLU02500
      E2=-C2/6.                                                                 FLU02510
      E3=-C3/6.                                                                 FLU02520
       F1=B1/2520.                                                              FIX00010
       F2=B2/2520.                                                              FIX00020
       F3=B3/2520.                                                              FIX00030
       G1=C1/2520.                                                              FIX00040
       G2=C2/2520.                                                              FIX00050
       G3=C3/2520.                                                              FIX00060
      U1=T1(N1)                                                                 FIX00080
      U2=T1(N2)                                                                 FIX00090
      U3=T1(N3)                                                                 FIX00100
      U4=T1(N4)                                                                 FIX00110
      U5=T1(N5)                                                                 FIX00120
      U6=T1(N6)                                                                 FIX00130
      V1=T1(N7)                                                                 FIX00140
      V2=T1(N8)                                                                 FIX00150
      V3=T1(N9)                                                                 FIX00160
      V4=T1(N10)                                                                FIX00170
      V5=T1(N11)                                                                FIX00180
      V6=T1(N12)                                                                FIX00190
      TM$(1,1)=.75 *(B1*B1+C1*C1)*CONST                                         FLU02590
      TM$(1,2)= (B1*B2+C1*C2)*CONST                                             FLU02600
      TM$(1,3)= -TM$(1,2)*.25                                                   FLU02610
      TM$(1,4)= 0.                                                              FLU02620
      TM$(1,6)= (B1*B3+C1*C3)*CONST                                             FLU02630
      TM$(1,5)= - TM$(1,6)*.25                                                  FLU02640
                  TM$(3,3)= .75 *(B2*B2+C2*C2)*CONST                            FLU02650
                  TM$(3,4)= (B2*B3+ C2*C3)*CONST                                FLU02660
                  TM$(3,5)=  -TM$(3,4)*.25                                      FLU02670
      TM$(2,1)= TM$(1,2)                                                        FLU02680
      TM$(2,3) = TM$(1,2)                                                       FLU02690
       TM$(2,2)=8./3.*(TM$(1,1)+TM$(3,3))+2.*TM$(1,2)                           FLU02700
      TM$(2,4)= 2.*TM$(1,6) +TM$(3,4)+TM$(1,2)+4./3.*TM$(3,3)                   FLU02710
      TM$(2,5)= 0.                                                              FLU02720
      TM$(2,6)= TM$(1,6)+2.*TM$(3,4) +TM$(1,2)+ 4./3.*TM$(1,1)                  FLU02730
```

```
      TM$(3,1)=TM$(1,3)                                                 FLU02740
      TM$(3,2)=TM$(2,3)                                                 FLU02750
      TM$(3,6)=0.                                                       FLU02760
                        TM$(5,5)=.75 *(B3*B3+C3*C3)*CONST               FLU02770
      TM$(4,1)=TM$(1,4)                                                 FLU02780
      TM$(4,2)=TM$(2,4)                                                 FLU02790
      TM$(4,3)=  TM$(3,4)                                               FLU02800
      TM$(4,4)=  8./3.*(TM$(3,3)+TM$(5,5))+2.*TM$(3,4)                  FLU02810
      TM$(4,5)=  TM$(3,4)                                               FLU02820
      TM$(4,6)=  TM$(1,6)+TM$(3,4)+2.*TM$(1,2)+4./3.*TM$(5,5)           FLU02830
      TM$(6,3)=TM$(3,6)                                                 FLU02840
      TM$(5,1)=  TM$(1,5)                                               FLU02850
      TM$(5,2)=  TM$(2,5)                                               FLU02860
      TM$(5,3)=  TM$(3,5)                                               FLU02870
      TM$(5,4)=  TM$(4,5)                                               FLU02880
      TM$(5,6)=  TM$(1,6)                                               FLU02890
      TM$(6,1)=TM$(1,6)                                                 FLU02900
      TM$(6,2)=TM$(2,6)                                                 FLU02910
      TM$(6,4)=TM$(4,6)                                                 FLU02920
      TM$(6,5)=TM$(5,6)                                                 FLU02930
      TM$(6,6)=  8./3.*(TM$(5,5)+TM$(1,1))+2.*TM$(1,6)                  FLU02940
       IF(NCASE.NE.1)GO TO 3000                                         FLU02950
C
C     BEGIN INPUT OF NON-LINEAR TERMS
C
       TM$(1,1)=TM$(1,1)-                                               FIX00190
     1 (F1*(78.*U1+48.*U2+(-9.*U3)+12.*U4+(-9.*U5)+48.*U6))             FIX00200
       TM$(1,2)=TM$(1,2)-                                               FIX00210
     1 (F2*(120.*U1+48.*U2-16.*U3-16.*U4-16.*U5+48.*U6))-               FIX00220
     2 (F1*(24.*U1-32.*U2-16.*U3-48.*U4+4.*U5-16.*U6))                  FIX00230
       TM$(1,3)=TM$(1,3)-                                               FIX00240
     1 (F2*(-18.*U1-32.*U2-9.*U3-20.*U4+11.*U5-16.*U6))                 FIX00250
       TM$(1,4)=TM$(1,4)-                                               FIX00260
     1 (F3*(24.*U1-32.*U2-16.*U3-48.*U4+4.*U5-16.*U6))-                 FIX00270
     2 (F2*(24.*U1-16.*U2+4.*U3-48.*U4-16.*U5-32.*U6))                  FIX00280
       TM$(1,5)=TM$(1,5)-                                               FIX00290
     1 (F3*(-18.*U1-16.*U2+11.*U3-20.*U4-9.*U5-32.*U6))                 FIX00300
       TM$(1,6)=TM$(1,6)-                                               FIX00310
     1 (F3*(120.*U1+48.*U2-16.*U3-16.*U4-16.*U5+48.*U6))-               FIX00320
     2 (F1*(24.*U1-16.*U2+4.*U3-48.*U4-16.*U5-32.*U6))                  FIX00330
       TM$(2,1)=TM$(2,1)-                                               FIX00340
     1 (F1*(48.*U1+160.*U2-32.*U3+16.*U4-20.*U5+80.*U6))                FIX00350
       TM$(2,2)=TM$(2,2)-                                               FIX00370
     1 (F2*(48.*U1+384.*U2-32.*U3+128.*U4-48.*U5+192.*U6))-             FIX00380
     2 (F1*(-32.*U1+384.*U2+48.*U3+192.*U4-48.*U5+128.*U6))             FIX00390
       TM$(2,3)=TM$(2,3)-                                               FIX00400
```

```
1  (F2*(-32.*U1+160.*U2+48.*U3+80.*U4-20.*U5+16.*U6))                    FIX00410
   TM$(2,4)=TM$(2,4)-                                                    FIX00420
1  (F3*(-32.*U1+384.*U2+48.*U3+192.*U4-48.*U5+128.*U6))-                 FIX00430
2  (F2*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))                  FIX00440
   TM$(2,5)=TM$(2,5)-                                                    FIX00450
1  (F3*(-16.*U1-96.*U2-16.*U3+16.*U4+12.*U5+16.*U6))                     FIX00460
   TM$(2,6)=TM$(2,6)-                                                    FIX00470
1  (F3*(48.*U1+384.*U2-32.*U3+128.*U4-48.*U5+192.*U6))-                  FIX00480
2  (F1*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))                  FIX00490
   TM$(3,1)=TM$(3,1)-                                                    FIX00500
1  (F1*(-9.*U1-32.*U2-18.*U3-16.*U4+11.*U5-20.*U6))                      FIX00510
   TM$(3,2)=TM$(3,2)-                                                    FIX00520
1  (F2*(-16.*U1-32.*U2+24.*U3-16.*U4+4.*U5-48.*U6))-                     FIX00530
2  (F1*(-16.*U1+48.*U2+120.*U3+48.*U4-16.*U5-16.*U6))                    FIX00540
   TM$(3,3)=TM$(3,3)-                                                    FIX00550
1  (F2*(-9.*U1+48.*U2+78.*U3+48.*U4-9.*U5+12.*U6))                       FIX00560
   TM$(3,4)=TM$(3,4)-                                                    FIX00570
1  (F3*(-16.*U1+48.*U2+120.*U3+48.*U4-16.*U5-16.*U6))-                   FIX00580
2  (F2*(4.*U1-16.*U2+24.*U3-32.*U4-16.*U5-48.*U6))                       FIX00590
   TM$(3,5)=TM$(3,5)-                                                    FIX00600
1  (F3*(11.*U1-16.*U2-18.*U3-32.*U4-9.*U5-20.*U6))                       FIX00610
   TM$(3,6)=TM$(3,6)-                                                    FIX00620
1  (F3*(-16.*U1-32.*U2+24.*U3-16.*U4+4.*U5-48.*U6))-                     FIX00630
2  (F1*(4.*U1-16.*U2+24.*U3-32.*U4-16.*U5-48.*U6))                       FIX00640
   TM$(4,1)=TM$(4,1)-                                                    FIX00650
1  (F1*(12.*U1+16.*U2-16.*U3-96.*U4-16.*U5+16.*U6))                      FIX00660
   TM$(4,2)=TM(4,2)-                                                     FIX00670
1  (F2*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))-                  &
2  (F1*(-48.*U1+192.*U2+48.*U3+384.*U4-32.*U5+128.*U6))                  FIX00690
   TM$(4,3)=TM$(4,3)-                                                    FIX00700
1  (F2*(-20.*U1+80.*U2+48.*U3+160.*U4-32.*U5+16.*U6))                    FIX00710
   TM$(4,4)=TM$(4,4)-                                                    FIX00720
1  (F3*(-48.*U1+192.*U2+48.*U3+384.*U4-32.*U5+128.*U6))-                 FIX00730
2  (F2*(-48.*U1+128.*U2-32.*U3+384.*U4+48.*U5+192.*U6))                  FIX00740
   TM$(4,5)=TM$(4,5)-                                                    FIX00750
1  (F3*(-20.*U1+16.*U2-32.*U3+160.*U4+48.*U5+80.*U6))                    FIX00760
   TM$(4,6)=TM$(4,6)-                                                    FIX00770
1  (F3*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))-                 FIX00780
2  (F1*(-48.*U1+128.*U2-32.*U3+384.*U4+48.*U5+192.*U6))                  FIX00790
   TM$(5,1)=TM$(5,1)-                                                    FIX00800
1  (F1*(-9.*U1-20.*U2+11.*U3-16.*U4-18.*U5-32.*U6))                      FIX00810
   TM$(5,2)=TM$(5,2)-                                                    FIX00820
1  (F2*(-16.*U1-48.*U2+4.*U3-16.*U4+24.*U5-32.*U6))-                     FIX00830
2  (F1*(4.*U1-48.*U2-16.*U3-32.*U4+24.*U5-16.*U6))                       FIX00840
   TM$(5,3)=TM$(5,3)-                                                    FIX00850
1  (F2*(11.*U1-20.*U2-9.*U3-32.*U4-18.*U5-16.*U6))                       FIX00860
   TM$(5,4)=TM$(5,4)-                                                    FIX00870
1  (F3*(4.*U1-48.*U2-16.*U3-32.*U4+24.*U5-16.*U6))-                      FIX00880
```

```
     2 (F2*(-16.*U1-16.*U2-16.*U3+48.*U4+120.*U5+48.*U6))                    FIX00890
       TM$(5,5)=TM$(5,5)-                                                   FIX00900
     1 (F3*(-9.*U1+12.*U2-9.*U3+48.*U4+78.*U5+48.*U6))                      F9X00   0
       TM$(5,6)=TM$(5,6)-                                                   FIX00920
     1 (F3*(-16.*U1-48.*U2+4.*U3-16.*U4+24.*U5-32.*U6))-                    FIX00930
     2 (F1*(-16.*U1-16.*U2-16.*U3+48.*U4+120.*U5+48.*U6))                   FIX00940
       TM$(6,1)=TM$(6,1)-                                                   FIX00950
     1 (F1*(48.*U1+80.*U2-20.*U3+16.*U4-32.*U5+160.*U6))                    FIX00960
       TM$(6,2)=TM$(6,2)-                                                   FIX00970
     1 (F2*(48.*U1+192.*U2-48.*U3+128.*U4-32.*U5+384.*U6))-                 FIX00980
     2 (F1*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))                 FIX00990
       TM$(6,3)=TM$(6,3)-                                                   FIX01000
     1 (F2*(-16.*U1+16.*U2+12.*U3+16.*U4-16.*U5-96.*U6))                    FIX01010
       TM$(6,4)=TM$(6,4)-                                                   FIX01020
     1 (F3*(-16.*U1+128.*U2-16.*U3+128.*U4-16.*U5+128.*U6))-                FIX01030
     2 (F2*(-32.*U1+128.*U2-48.*U3+192.*U4+48.*U5+384.*U6))                 FIX01040
       TM$(6,5)=TM$(6,5)-                                                   FIX01050
     1 (F3*(-32.*U1+16.*U2-20.*U3+80.*U4+48.*U5+160.*U6))                   FIX01060
       TM$(6,6)=TM$(6,6)-                                                   FIX01070
     1 (F3*(48.*U1+192.*U2-48.*U3+128.*U4-32.*U5+384.*U6))-                 FIX01080
     2 (F1*(-32.*U1+128.*U2-48.*U3+192.*U4+48.*U5+384.*U6))                 FIX01090
C                                                                           FIX01100
C                                                                           FIX01110
       TM$(1,1)=TM$(1,1)-                                                   FIX00200
    1(G1*(78.*V1+48.*V2+(-9.*V3)+12.*V4+(-9.*V5)+48.*V6))                   FIX00210
       TM$(1,2)=TM$(1,2)-                                                   FIX00220
    1(G2*(120.*V1+48.*V2-16.*V3-16.*V4-16.*V5+48.*V6))-                     FIX00230
    2(G1*(24.*V1-32.*V2-16.*V3-48.*V4+4.*V5-16.*V6))                        FIX00240
       TM$(1,3)=TM$(1,3)-                                                   FIX00250
    1(G2*(-18.*V1-32.*V2-9.*V3-20.*V4+11.*V5-16.*V6))                       FIX00260
       TM$(1,4)=TM$(1,4)-                                                   FIX00270
    1(G3*(24.*V1-32.*V2-16.*V3-48.*V4+4.*V5-16.*V6))-                       FIX00280
    2(G2*(24.*V1-16.*V2+4.*V3-48.*V4-16.*V5-32.*V6))                        FIX00290
       TM$(1,5)=TM$(1,5)-                                                   FIX00300
    1(G3*(-18.*V1-16.*V2+11.*V3-20.*V4-9.*V5-32.*V6))                       FIX00310
       TM$(1,6)=TM$(1,6)-                                                   FIX00320
    1(G3*(120.*V1+48.*V2-16.*V3-16.*V4-16.*V5+48.*V6))-                     FIX00330
    2(G1*(24.*V1-16.*V2+4.*V3-48.*V4-16.*V5-32.*V6))                        FIX00340
       TM$(2,1)=TM$(2,1)-                                                   FIX00350
    1(G1*(48.*V1+160.*V2-32.*V3+16.*V4-20.*V5+80.*V6))                      FIX00360
                                                                            FIX00370
       TM$(2,2)=TM$(2,2)-                                                   FIX00380
    1(G2*(48.*V1+384.*V2-32.*V3+128.*V4-48.*V5+192.*V6))-                   FIX00390
    2(G1*(-32.*V1+384.*V2+48.*V3+192.*V4-48.*V5+128.*V6))                   FIX00400
       TM$(2,3)=TM$(2,3)-                                                   FIX00410
    1(G2*(-32.*V1+160.*V2+48.*V3+80.*V4-20.*V5+16.*V6))                     FIX00420
       TM$(2,4)=TM$(2,4)-                                                   FIX00430
    1(G3*(-32.*V1+384.*V2+48.*V3+192.*V4-48.*V5+128.*V6))-                  FIX00440
```

```
2(G2*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))                    FIX00450
  TM$(2,5)=TM$(2,5)-                                                     FIX00460
1(G3*(-16.*V1-96.*V2-16.*V3+16.*V4+12.*V5+16.*V6))                       FIX00470
  TM$(2,6)=TM$(2,6)-                                                     FIX00480
1(G3*(48.*V1+384.*V2-32.*V3+128.*V4-48.*V5+192.*V6))-                    FIX00490
2(G1*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))                    FIX00500
  TM$(3,1)=TM$(3,1)-                                                     FIX00510
1(G1*(-9.*V1-32.*V2-18.*V3-16.*V4+11.*V5-20.*V6))                        FIX00520
  TM$(3,2)=TM$(3,2)-                                                     FIX00530
1(G2*(-16.*V1-32.*V2+24.*V3-16.*V4+4.*V5-48.*V6))-                       FIX00540
2(G1*(-16.*V1+48.*V2+120.*V3+48.*V4-16.*V5-16.*V6))                      FIX00550
  TM$(3,3)=TM$(3,3)-                                                     FIX00560
1(G2*(-9.*V1+48.*V2+78.*V3+48.*V4-9.*V5+12.*V6))                         FIX00570
  TM$(3,4)=TM$(3,4)-                                                     FIX00580
1(G3*(-16.*V1+48.*V2+120.*V3+48.*V4-16.*V5-16.*V6))-                     FIX00590
2(G2*(4.*V1-16.*V2+24.*V3-32.*V4-16.*V5-48.*V6))                         FIX00600
  TM$(3,5)=TM$(3,5)-                                                     FIX00610
1(G3*(11.*V1-16.*V2-18.*V3-32.*V4-9.*V5-20.*V6))                         FIX00620
  TM$(3,6)=TM$(3,6)-                                                     FIX00630
1(G3*(-16.*V1-32.*V2+24.*V3-16.*V4+4.*V5-48.*V6))-                       FIX00640
2(G1*(4.*V1-16.*V2+24.*V3-32.*V4-16.*V5-48.*V6))                         FIX00650
  TM$(4,1)=TM$(4,1)-                                                     FIX00660
1(G1*(12.*V1+16.*V2-16.*V3-96.*V4-16.*V5+16.*V6))                        FIX00670
  TM$(4,2)=TM(4,2)-                                                      FIX00680
1(G2*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))-                   FIX00690
2(G1*(-48.*V1+192.*V2+48.*V3+384.*V4-32.*V5+128.*V6))                    FIX00700
  TM$(4,3)=TM$(4,3)-                                                     FIX00710
1(G2*(-20.*V1+80.*V2+48.*V3+160.*V4-32.*V5+16.*V6))                      FIX00720
  TM$(4,4)=TM$(4,4)-                                                     FIX00730
1(G3*(-48.*V1+192.*V2+48.*V3+384.*V4-32.*V5+128.*V6))-                   FIX00740
2(G2*(-48.*V1+128.*V2-32.*V3+384.*V4+48.*V5+192.*V6))                    FIX00750
  TM$(4,5)=TM$(4,5)-                                                     FIX00760
1(G3*(-20.*V1+16.*V2-32.*V3+160.*V4+48.*V5+80.*V6))                      FIX00770
  TM$(4,6)=TM$(4,6)-                                                     FIX00780
1(G3*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))-                   FIX00790
2(G1*(-48.*V1+128.*V2-32.*V3+384.*V4+48.*V5+192.*V6))                    FIX00800
  TM$(5,1)=TM$(5,1)-                                                     FIX00810
1(G1*(-9.*V1-20.*V2+11.*V3-16.*V4-18.*V5-32.*V6))                        FIX00820
  TM$(5,2)=TM$(5,2)-                                                     FIX00830
1(G2*(-16.*V1-48.*V2+4.*V3-16.*V4+24.*V5-32.*V6))-                       FIX00840
2(G1*(4.*V1-48.*V2-16.*V3-32.*V4+24.*V5-16.*V6))                         FIX00850
  TM$(5,3)=TM$(5,3)-                                                     FIX00860
1(G2*(11.*V1-20.*V2-9.*V3-32.*V4-18.*V5-16.*V6))                         FIX00870
  TM$(5,4)=TM$(5,4)-                                                     FIX00880
1(G3*(4.*V1-48.*V2-16.*V3-32.*V4+24.*V5-16.*V6))-                        FIX00890
2(G2*(-16.*V1-16.*V2-16.*V3+48.*V4+120.*V5+48.*V6))                      FIX00900
  TM$(5,5)=TM$(5,5)-                                                     FIX00910
1(G3*(-9.*V1+12.*V2-9.*V3+48.*V4+78.*V5+48.*V6))                         FIX00920
```

```
      TM$(5,6)=TM$(5,6)-                                                FIX00930
     1(G3*(-16.*V1-48.*V2+4.*V3-16.*V4+24.*V5-32.*V6))-                 FIX00940
     2(G1*(-16.*V1-16.*V2-16.*V3+48.*V4+120.*V5+48.*V6))                FIX00950
      TM$(6,1)=TM$(6,1)-                                                FIX00960
     1(G1*(48.*V1+80.*V2-20.*V3+16.*V4-32.*V5+160.*V6))                 FIX00970
      TM$(6,2)=TM$(6,2)-                                                FIX00980
     1(G2*(48.*V1+192.*V2-48.*V3+128.*V4-32.*V5+384.*V6))-              697   99
     2(G1*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))              FIX01000
      TM$(6,3)=TM$(6,3)-                                                FIX01010
     1(G2*(-16.*V1+16.*V2+12.*V3+16.*V4-16.*V5-96.*V6))                 FIX01020
      TM$(6,4)=TM$(6,4)-                                                FIX01030
     1(G3*(-16.*V1+128.*V2-16.*V3+128.*V4-16.*V5+128.*V6))-             FIX01040
     2(G2*(-32.*V1+128.*V2-48.*V3+192.*V4+48.*V5+384.*V6))              FIX01050
      TM$(6,5)=TM$(6,5)-                                                FIX01060
     1(G3*(-32.*V1+16.*V2-20.*V3+80.*V4+48.*V5+160.*V6))                FIX01070
      TM$(6,6)=TM$(6,6)-                                                FIX01080
     1(G3*(48.*V1+192.*V2-48.*V3+128.*V4-32.*V5+384.*V6))-              FIX01090
     2(G1*(-32.*V1+128.*V2-48.*V3+192.*V4+48.*V5+384.*V6))              FIX01100
C                                                                       FIX01110
CCC                                                                     FIX01120
CCC          THIS ENDS THE ADDITION OF NON-LINEAR TERMS TO              FIX01120
CCC              THE LOCAL ARRAY.                                       FIX01140
C                                                                       FIX01150
 3000  CONTINUE                                                         FLU03380
      TM$(7,7)=TM$(1,1)                                                 FLU03390
      TM$(7,8)=TM$(1,2)                                                 FLU03400
      TM$(7,9)=TM$(1,3)                                                 FLU03410
      TM$(7,10)=TM$(1,4)                                                FLU03420
      TM$(7,11)=TM$(1,5)                                                FLU03430
      TM$(7,12)=TM$(1,6)                                                FLU03440
      TM$(8,7)=TM$(2,1)                                                 FLU03450
      TM$(8,8)=TM$(2,2)                                                 FLU03460
      TM$(8,9)=TM$(2,3)                                                 FLU03470
      TM$(8,10)=TM$(2,4)                                                FLU03480
      TM$(8,11)=TM$(2,5)                                                FLU03490
      TM$(8,12)=TM$(2,6)                                                FLU03500
      TM$(9,7)=TM$(3,1)                                                 FLU03510
      TM$(9,8)=TM$(3,2)                                                 FLU03520
      TM$(9,9)=TM$(3,3)                                                 FLU03530
      TM$(9,10)=TM$(3,4)                                                FLU03540
      TM$(9,11)=TM$(3,5)                                                FLU03550
      TM$(9,12)=TM$(3,6)                                                FLU03560
      TM$(10,7)=TM$(4,1)                                                FLU03570
      TM$(10,8)=TM$(4,2)                                                FLU03580
      TM$(10,9)=TM$(4,3)                                                FLU03590
      TM$(10,10)=TM$(4,4)                                               FLU03600
      TM$(10,11)=TM$(4,5)                                               FLU03610
      TM$(10,12)=TM$(4,6)                                               FLU03620
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```
TM$(11,7)=TM$(5,1)                                                    FLU03630
TM$(11,8)=TM$(5,2)                                                    FLU03640
TM$(11,9)=TM$(5,3)                                                    FLU03650
TM$(11,10)=TM$(5,4)                                                   FLU03660
TM$(11,11)=TM$(5,5)                                                   FLU03670
TM$(11,12)=TM$(5,6)                                                   FLU03680
TM$(12,7)=TM$(6,1)                                                    FLU03690
TM$(12,8)=TM$(6,2)                                                    FLU03700
TM$(12,9)=TM$(6,3)                                                    FLU03710
TM$(12,10)=TM$(6,4)                                                   FLU03720
TM$(12,11)=TM$(6,5)                                                   FLU03730
TM$(12,12)=TM$(6,6)                                                   FLU03740
TM$(13,1)= D1                                                         FLU03750
TM$(1,13)=TM$(13,1)                                                   FLU03760
TM$(14,1)=0.                                                          FLU03770
TM$(1,14)=TM$(14,1)                                                   FLU03780
TM$(15,1)=0.                                                          FLU03790
TM$(1,15)=TM$(15,1)                                                   FLU03800
TM$(13,2)= D1+ 2.*D2                                                  FLU03810
TM$(2,13)=TM$(13,2)                                                   FLU03820
TM$(14,2)=2.*D1+D2                                                    FLU03830
TM$(2,14)=TM$(14,2)                                                   FLU03840
TM$(15,2)= D1+D2                                                      FLU03850
TM$(2,15)=TM$(15,2)                                                   FLU03860
TM$(13,3)= 0.                                                         FLU03870
TM$(3,13)=TM$(13,3)                                                   FLU03880
TM$(14,3)=D2                                                          FLU03890
TM$(3,14)=TM$(14,3)                                                   FLU03900
TM$(15,3)= 0.                                                         FLU03910
TM$(3,15)=TM$(15,3)                                                   FLU03920
TM$(13,4)=D2+D3                                                       FLU03930
TM$(4,13)=TM$(13,4)                                                   FLU03940
TM$(14,4)=D2+2.*D3                                                    FLU03950
TM$(4,14)=TM$(14,4)                                                   FLU03960
TM$(15,4)=2.*D2+D3                                                    FLU03970
TM$(4,15)=TM$(15,4)                                                   FLU03980
TM$(13,5)=0.                                                          FLU03990
TM$(5,13)=TM$(13,5)                                                   FLU04000
TM$(14,5)=0.                                                          FLU04010
TM$(5,14)=TM$(14,5)                                                   FLU04020
TM$(15,5)=D3                                                          FLU04030
TM$(5,15)=TM$(15,5)                                                   FLU04040
TM$(13,6)=D1+2.*D3                                                    FLU04050
TM$(6,13)=TM$(13,6)                                                   FLU04060
TM$(14,6)=D1+D3                                                       FLU04070
TM$(6,14)=TM$(14,6)                                                   FLU04080
TM$(15,6)=2.*D1+D3                                                    FLU04090
TM$(6,15)=TM$(15,6)                                                   FLU04100
```

```
      TM$(13,7)=E1                                                      FLU04110
      TM$(7,13)=TM$(13,7)                                               FLU04120
      TM$(14,7)=0.                                                      FLU04130
      TM$(7,14)=TM$(14,7)                                               FLU04140
      TM$(15,7)=0.                                                      FLU04150
      TM$(7,15)=TM$(15,7)                                               FLU04160
      TM$(13,8)=E1+2.*E2                                                FLU04170
      TM$(8,13)=TM$(13,8)                                               FLU04180
      TM$(14,8)=2.*E1+E2                                                FLU04190
      TM$(8,14)=TM$(14,8)                                               FLU04200
      TM$(15,8)= E1+E2                                                  FLU04210
      TM$(8,15)=TM$(15,8)                                               FLU04220
      TM$(13,9)= 0.                                                     FLU04230
      TM$(9,13)=TM$(13,9)                                               FLU04240
      TM$(14,9)= E2                                                     FLU04250
      TM$(9,14)=TM$(14,9)                                               FLU04260
      TM$(15,9)= 0.                                                     FLU04270
      TM$(9,15)=TM$(15,9)                                               FLU04280
      TM$(13,10)=E2+E3                                                  FLU04290
      TM$(10,13)=TM$(13,10)                                             FLU04300
      TM$(14,10)=E2+2.*E3                                               FLU04310
       TM$(10,14)=TM$(14,10)                                            FLU04320
        TM$(15,10)=E2+E2+E3                                             FLU04330
      TM$(10,15)=TM$(15,10)                                             FLU04340
      TM$(13,11)= 0.                                                    FLU04350
      TM$(11,13)=TM$(13,11)                                             FLU04360
      TM$(14,11)=0.                                                     FLU04370
      TM$(11,14)=TM$(14,11)                                             FLU04380
      TM$(15,11)= E3                                                    FLU04390
      TM$(11,15)=TM$(15,11)                                             FLU04400
         TM$(13,12)=E1+E3+E3                                            FLU04410
      TM$(12,13)=TM$(13,12)                                             FLU04420
      TM$(14,12)=E1+E3                                                  FLU04430
      TM$(12,14)=TM$(14,12)                                             FLU04440
      TM$(15,12)=2.*E1+E3                                               FLU04450
      TM$(12,15)=TM$(15,12)                                             FLU04460
C
       CONST2=1/(4.*DEL)
      TM$(13,13)=-.5*(B1*B1+C1*C1) *CONST2
      TM$(13,14)=-.5*(B1*B2+C1*C2)*CONST2
      TM$(13,15)=-.5*(B1*B3+C1*C3)*CONST2
      TM$(14,13)=TM$(13,14)
      TM$(14,14)=-.5*(B2*B2+C2*C2)*CONST2
      TM$(14,15)=-.5*(B2*B3+C2*C3)*CONST2
      TM$(15,13)=TM$(13,15)
      TM$(15,14)=TM$(14,15)
      TM$(15,15)=-.5*(B3*B3+C3*C3)*CONST2
C                                                                       QZ 00170
```

```
C                                                                       QZ  00180
      U(1)=U1                                                           QZ  00190
      U(2)=U2                                                           QZ  00200
      U(3)=U3                                                           QZ  00210
      U(4)=U4                                                           QZ  00220
      U(5)=U5                                                           QZ  00230
      U(6)=U6                                                           QZ  00240
      V(1)=V1                                                           QZ  00250
      V(2)=V2                                                           QZ  00260
      V(3)=V3                                                           QZ  00270
      V(4)=V4                                                           QZ  00280
      V(5)=V5                                                           QZ  00290
      V(6)=V6                                                           QZ  00300
      NT(1)=N1
      NT(2)=N3
      NT(3)=N5
C                                                                       QZ  00340
C                                                                       QZ  00350
C                                                                       QZ  00360
      CONST3=1./DEL*4.*60.
      QZ1(1,1)=0.0                                                      QAR00030
      QZ1(2,1)=56.*(B2*C1-B1*C2)*CONST3                                 QAR00040
      QZ1(2,2)=0.0                                                      QAR00050
      QZ1(3,1)=-8.*(B2*C1-B1*C2)*CONST3                                 QAR00060
      QZ1(3,2)=12.*(B2*C1-B1*C2)*CONST3                                 QAR00070
      QZ1(3,3)=0.0                                                      QAR00080
      QZ1(4,1)=12.*(B3*C1+B2*C1-B1*C3-B1*C2)*CONST3                     QAR00090
      QZ1(4,2)=(16.*(B2*C1-B1*C2)+32.*(B3*C2+B3*C1-B2*C3-B1*C3))*CONST3 QAR00100
      QZ1(4,3)=12.*(B3*C2-B2*C3)*CONST3                                 QAR00110
      QZ1(4,4)=0.0                                                      QAR00120
      QZ1(5,1)=-8.*(B3*C1-B1*C3)*CONST3                                 QAR00130
      QZ1(5,2)=(-8.*(B3*C2-B2*C3)-4.*(B3*C1-B1*C3))*CONST3              QAR00140
      QZ1(5,3)=-4.*(B3*C2-B2*C3)*CONST3                                 QAR00150
      QZ1(5,4)=12.*(B3*C2-B2*C3)*CONST3                                 QAR00160
      QZ1(5,5)=0.0                                                      QAR00170
      QZ1(6,1)=56.*(B3*C1-B1*C3)*CONST3                                 QAR00180
      QZ1(6,2)=(96.*(B3*C2-B2*C3)+32.*(B1*C2+B3*C1-B2*C1-B1*C3))*CONST3 QAR00190
      QZ1(6,3)=(-4.*(B1*C2-B2*C1)-8.*(B3*C2-B2*C3))*CONST3              QAR00200
      QZ1(6,4)=(16.*(B1*C3-B3*C1)+32.*(B3*C2+B1*C2-B2*C3-B2*C1))*CONST3 QAR00210
      QZ1(6,5)=12.*(B1*C3-B3*C1)*CONST3                                 QAR00220
      QZ1(6,6)=0.0                                                      QAR00230
      QZ2(1,1)=0.0                                                      QAR00030
      QZ2(2,1)=12.*(B2*C1-B1*C2)*CONST3                                 QAR00040
      QZ2(2,2)=0.0                                                      QAR00050
      QZ2(3,1)=-8.*(B2*C1-B1*C2)*CONST3                                 QAR00060
      QZ2(3,2)=56.*(B2*C1-B1*C2)*CONST3                                 QAR00070
      QZ2(3,3)=0.0                                                      QAR00080
      QZ2(4,1)=(-8.*(B3*C1-B1*C3)-4.*(B2*C1-B1*C2))*CONST3              QAR00090
```

```
      QZ2(4,2)=(96.*(B3*C1-B1*C3)+32.*(B2*C1+B3*C2-B1*C2-B2*C3))*CONST3 QAR00100
      QZ2(4,3)=56.*(B3*C2-B2*C3)*CONST3                               QAR00110
      QZ2(4,4)=0.0                                                   QAR00120
      QZ2(5,1)=-4.*(B3*C1-B1*C3)*CONST3                              QAR00130
      QZ2(5,2)=(-4.*(B3*C2-B2*C3)-8.*(B3*C1-B1*C3))*CONST3           QAR00140
      QZ2(5,3)=-8.*(B3*C2-B2*C3)*CONST3                              QAR00150
      QZ2(5,4)=12.*(B3*C2-B2*C3)*CONST3                              QAR00160
      QZ2(5,5)=0.0                                                   QAR00170
      QZ2(6,1)=12.*(B3*C1-B1*C3)*CONST3                              QAR00180
      QZ2(6,2)=(16.*(B1*C2-B2*C1)+32.*(B3*C1+B3*C2-B1*C3-B2*C3))*CONST3 QAR00190
      QZ2(6,3)=12.*(B1*C2+B3*C2-B2*C1-B2*C3)*CONST3                  QAR00200
      QZ2(6,4)=(16.*(B3*C2-B2*C3)+32.*(B1*C3+B1*C2-B3*C1-B2*C1))*CONST3 QAR00210
      QZ2(6,5)=12.*(B1*C3-B3*C1)*CONST3                              QAR00220
      QZ2(6,6)=0.0                                                   QAR00230
      QZ3(1,1)=0.0                                                   QAR00030
      QZ3(2,1)=12.*(B2*C1-B1*C2)*CONST3                              QAR00040
      QZ3(2,2)=0.0                                                   QAR00050
      QZ3(3,1)=-4.*(B2*C1-B1*C2)*CONST3                              QAR00060
      QZ3(3,2)=12.*(B2*C1-B1*C2)*CONST3                              QAR00070
      QZ3(3,3)=0.0                                                   QAR00080
      QZ3(4,1)=(-4.*(B3*C1-B1*C3)-8.*(B2*C1-B1*C2))*CONST3           QAR00090
       QZ3(4,2)=(16.*(B3*C2-B2*C3)+32.*(B2*C1+B3*C1-B1*C2-B1*C3))*CONST3QAR00100
      QZ3(4,3)=12.*(B3*C2-B2*C3)*CONST3                              QAR00110
      QZ3(4,4)=0.0                                                   QAR00120
      QZ3(5,1)=-8.*(B3*C1-B1*C3)*CONST3                              QAR00130
      QZ3(5,2)=(12.*(B3*C2-B2*C3+B3*C1-B1*C3))*CONST3                QAR00140
      QZ3(5,3)=-8.*(B3*C2-B2*C3)*CONST3                              QAR00150
      QZ3(5,4)=56.*(B3*C2-B2*C3)*CONST3                              QAR00160
      QZ3(5,5)=0.0                                                   QAR00170
      QZ3(6,1)=12.*(B3*C1-B1*C3)*CONST3                              QAR00180
      QZ3(6,2)=(16.*(B3*C1-B1*C3)+32.*(B1*C2+B3*C2-B2*C1-B2*C3))*CONST3 QAR00190
      QZ3(6,3)=(-8.*(B1*C2-B2*C1)-4.*(B3*C2-B2*C3))*CONST3           QAR00200
      QZ3(6,4)=(96.*(B1*C2-B2*C1)+32.*(B3*C2+B1*C3-B2*C3-B3*C1))*CONST3
      QZ3(6,5)=56.*(B1*C3-B3*C1)*CONST3                              QAR00220
      QZ3(6,6)=0.0                                                   QAR00230
C                                                                    QZ  00380
C                                                                    QZ  00390
       DO 57 JA=1,14
       DO 57 JB=1,3                                                  QZ  00410
       ICHECK=NT(JB)-NI(JA)
       IF(ICHECK.EQ.0) GO TO 58
       GO TO 57
   58  IF(JB-2)60,70,80
   60  QPRM=0.0                                                      QZ  00470
       DO 61 KO=1,6                                                  QZ  00480
       DO 61 KM=1,KO                                                 QZ  00490
      QPRM=QPRM+(U(KO)*V(KM)-U(KM)*V(KO))*QZ1(KO,KM)                 QZ  00510
   61  CONTINUE                                                      QZ  00520
```

```
      Q(NCP(NT(JB)))=QPRM+Q(NCP(NT(JB)))
       GO TO 57
   70  QPRM=0.0                                                          QZ 00550
       DO 71 KN=1,6                                                      QZ 00560
       DO 71 KL=1,KN                                                     QZ 00570
      QPRM=QPRM+(U(KN)*V(KL)-U(KL)*V(KN))*QZ2(KN,KL)                     QZ 00580
   71  CONTINUE                                                          QZ 00590
      Q(NCP(NT(JB)))=QPRM+Q(NCP(NT(JB)))
       GO TO 57
   80 QPRM=0.0
       DO 81 KP=1,6                                                      QZ 00630
       DO 81 KQ=1,6                                                      QZ 00640
       QPRM=QPRM+(U(KP)*V(KQ)-U(KQ)*V(KP))*QZ3(KP,KQ)                    QZ 00650
   81 CONTINUE
      Q(NCP(NT(JB)))=QPRM+Q(NCP(NT(JB)))
   57 CONTINUE
      QZC(1)=C1*C1
      QZC(2)=2.*C1*C2
      QZC(3)=C2*C2
      QZC(4)=2.*C2*C3
      QZC(5)=C3*C3
      QZC(6)=2.*C1*C3
      QZB(1)=B1*B1
      QZB(2)=2.*B1*B2
      QZB(3)=B2*B2
      QZB(4)=2.*B2*B3
      QZB(5)=B3*B3
      QZB(6)=2.*C1*C3
      NLCY(1)=17                                                         GO 00020
      NLCY(2)=18                                                         GO 00030
      NLCY(3)=20                                                         GO 00040
      NLCY(4)=22                                                         GO 00050
      NRCY(1)=30                                                         GO 00060
      NRCY(2)=33                                                         GO 00070
      NRCY(3)=34                                                         GO 00080
      NRCY(4)=36                                                         GO 00090
      CONST4=A/(4.*DEL**2)                                               GO 00100
      DO 513 LA=1,4                                                      GO 00110
      JCHECK=K-NLCY(LA)                                                  GO 00120
      KCHECK=K-NRCY(LA)                                                  GO 00130
      IF(JCHECK.EQ.0) GO TO 517                                          GO 00140
      IF(KCHECK.EQ.0) GO TO 519                                          GO 00150
      GO TO 513                                                          GO 00160
  517 THETA=ATAN((YC(NT(3))-YC(NT(2)))/(XC(NT(3))-XC(NT(2))))            GO 00170
      THETA1=THETA-2.*ATAN(1.)                                           GO 00180
      SLIJ=SQRT((YC(NT(3))-YC(NT(2)))**2+(XC(NT(3))-XC(NT(2)))**2)
      XHAT=COS(THETA1)
      YHAT=SIN(THETA1)
```

```
      PART1=0.0
      PART2=0.0
      TPART=0.0
      DO 523 LB=1,6
      PART1=PART1+(U(LB)*QZB(LB))*CONST4
      PART2=PART2+(V(LB)*QZC(LB))*CONST4
  523 CONTINUE
      TPART=PART1+PART2
      Q(NCP(NT(2)))=Q(NCP(NT(2)))+TPART*SLIJ
      Q(NCP(NT(3)))=Q(NCP(NT(3)))+TPART*SLIJ
      GO TO 513
  519 THETA2=ATAN((YC(NT(1))-YC(NT(2)))/(XC(NT(2))-XC(NT(1))))          GO 00210
      THETA1=6.*ATAN(1.)-THETA2                                         GO 00220
      SLIJ=SQRT((YC(NT(1))-YC(NT(2)))**2+(XC(NT(1))-XC(NT(2)))**2)
      XHAT=COS(THETA1)                                                  GO 00240
      YHAT=SIN(THETA1)                                                  GO 00250
      TPART=0.0
      PART1=0.0
      PART2=0.0
      DO 521 LB=1,6                                                     GO 00260
      PART1=PART1+(U(LB)*QZB(LB))*CONST4                                GO 00270
      PART2=PART2+(V(LB)*QZC(LB))*CONST4                                GO 00280
  521 CONTINUE                                                          GO 00290
      TPART=PART1+PART2                                                 GO 00300
      Q(NCP(NT(1)))=Q(NCP(NT(1)))+TPART*SLIJ
      Q(NCP(NT(2)))=Q(NCP(NT(2)))+TPART*SLIJ
  513 CONTINUE                                                          GO 00320
  185 CONTINUE                                                          FLU04470
      N(1)= N1                                                          FLU04480
      N(2)= N2                                                          FLU04490
      N(3)= N3                                                          FLU04500
       N(4)=N4                                                          FLU04510
       N(5)=N5                                                          FLU04520
       N(6)=N6                                                          FLU04530
      N(7) =N7                                                          FLU04540
      N(8) =N8                                                          FLU04550
      N(9) =N9                                                          FLU04560
      N(10)=N10                                                         FLU04570
      N(11)=N11                                                         FLU04580
      N(12)=N12                                                         FLU04590
      N(13)=N13                                                         FLU04600
      N(14)=N14                                                         FLU04610
      N(15)=N15                                                         FLU04620
      DO 200  I$=1,15                                                   FLU04630
      I= N(I$)                                                          FLU04640
      DO 200  J$=1,15                                                   FLU04650
      J=N(J$)                                                           FLU04660
      TM(I,J)=TM(I,J)+TM$(I$,J$)                                        FLU04670
```

```
  200 CONTINUE                                                          FLU04680
  300 CONTINUE                                                          FLU04690
      IF(NNQXY.EQ.0) GO TO 310                                          FLU04700
      DO 310  I=1,NNQXY                                                 FLU04710
      RHS(NQS(I))=RHS(NQS(I))+Q(NQS(I))                                 FLU04720
        RHS(NQS(I)+NN)=RHS(NQS(I)+NN)+Q(NQS(I)+NN)                      FLU04730
      IF(I-NNPQ)309,309,310
  309 RHS(NCP(NPS(I+NNPS)))=RHS(NCP(NPS(I+NNPS)))+Q(NCP(NPS(I+NNPS)))
  310 CONTINUE                                                          FLU04740
  320 CONTINUE                                                          FLU04750
      CALL SOLMIX(MM,TM,T,RHS,NTOTVP,NVIS,NTOTQ,NQIS,B,Z,C)             FLU04760
      WRITE(NWRITE,2000)                                                FLU04770
      NTEST=0
       DO 322 J=1,MM                                                    FLU04780
       ESPILA=ABS(T1(J)-T(J))                                           FLU04790
       IF(ESPILA-.001)322,324,324                                       FLU04800
  322 CONTINUE                                                          FLU04810
      NTEST=100
  324 CCNTINUE                                                          FLU04820
      DO 325 I=1,NN
      IV=I+NN
      IQ=I+2*NN
      T1(I)=            T(I)
      Q(I)=RHS(I)
      T1(IV)=             T(IV)
      Q(IV)=RHS(IV)
      IF(MM-IQ)2223,2222,2222
 2222 T1(IQ)=             T(IQ)
      Q(IQ)=RHS(IQ)
      WRITE(NWRITE,2300)I,T(I),IV,T(IV),IQ,T(IQ),Q(I),Q(IV),Q(IQ)
      GO TO 325
 2223 WRITE(NWRITE,2301)I,T(I),IV,T(IV),Q(I),Q(IV)
  325 CONTINUE                                                          FLU04860
      IF(NTEST.NE.100) GO TO 177
  500 FORMAT(I10)                                                       FLU04880
 1005 FORMAT(3I10)                                                      FLU04890
 1006 FORMAT(6X,A4,I10,2F10.0)                                          FLU04900
 1010 FORMAT(7I10)                                                      FLU04910
 1015 FORMAT(6X,A4,I10,F10.0)                                           FLU04920
 1016 FORMAT(6X,A4,I10)                                                 FLU04930
 1020 FORMAT(I10,2F10.0)                                                FLU04940
 1025 FORMAT(6X,A4,I10,F10.0)                                           FLU04950
 1030 FORMAT(6X,A4,2I10,2F10.0)                                         FLU04960
 1035 FORMAT(5X,'NO. OF NODES=',I3,//,5X,'NO. OF ELEMENTS=',I3,//,      FLU04970
     15X,'NO. OF CORNER NODES=',I3,//)                                  FLU04980
 1040 FORMAT(5X,'SUMMARY OF NODAL COORDINATES',//,                      FLU04990
     17X,'I',12X'X(I)',12X,'Y(I)',//)                                   FLU05000
 1045 FORMAT(5X,I3,2(7X,F10.3))                                         FLU05010
```

```
1050 FORMAT(5X,'LISTING OF SYSTEM TOPOLOGY'//,5X                          FLU05020
     1'ELEMENT NUMBER',20X,'NODE NUMBERS',//)                            FLU05030
1055 FORMAT(5X,I3,10X,6(5X,I3))                                          FLU05040
1060 FORMAT(7X,'NODES WHERE VELOCITIES ARE SPECIFIED',                   FLU05050
     1//,8X,'I',5X,'NODE',5X,'U VELOCITY',5X,'V VELOCITY',//)            FLU05060
1065 FORMAT(2X,2(4X,I3),3X,F12.3,3X,F12.3)                               FLU05070
1070 FORMAT(5X,'SPECIFIED RIGHT HAND SIDE I.E. QX AND QY',               FLU05080
     1      //,2X,'              NODE',5X,'          QX',5X,'   QY',//)  FLU05090
1071 FORMAT(5X,'SPECIFIED RIGHT HAND SIDE FOR PRESSURE NODES, I.E. QZ'   PRE00200
     1,//,2X,'              NODE',5X,'            QZ',//)                PRE00210
1072 FORMAT(2X,2(4X,I3),3X,F12.3)                                        PRE00220
1075 FORMAT(5X,I3,2(10X,F12.3))                                          FLU05100
1080 FORMAT(5X,'              NODES WHERE PRESSURE IS SPECIFIED',        FLU05110
     1          //,5X,'I',5X,'NODE',10X,'PRESSURE',//)                  FLU05120
1085 FORMAT(5X,I3,3X,I3,10X,F12.3)                                       FLU05130
2300 FORMAT(6X,3(I3,5X,E13.5,5X),3(E13.5,3X))
2301 FORMAT(6X,2(I3,5X,E13.5,5X),26X,2(E13.5,3X))
2000 FORMAT(3X,'NODE NO.',3X,'T(J)  X-COORD',5X,'NODE NO.',2X,'T(J)',2
     1X,'Y-COORD',5X,'NODE NO.',4X,'T(I)',9X,'Q(I) X',17X,'Q(I) Y',12
     2 X,'Q(I) P')
2020 FORMAT(1H1,5X,'THIS IS A NON-LINEAR PROBLEM',//)                     FLU05210
      STOP                                                               FLU05220
      END                                                                FLU05230
```

```
      SUBROUTINE SOLMIX(N,A,X,Y,NX,LISTX,NY,LISTY,B,Z,C)                SOL00010
      DIMENSION A(N,N),X(N),Y(N),LISTX(N),LISTY(N)                      SOL00020
      DIMENSION  B(NY,NY),Z(NY),C(NY)                                   SOL00030
      IF(N.EQ.(NX+NY)) GO TO 100                                        SOL00040
      WRITE(6,111) N,NX,NY                                              SOL00050
  111 FORMAT(10X,'N=',I2,10X,'NX=',I2,10X,'NY=',I2)                     SOL00060
      STOP                                                              SOL00070
  100 CONTINUE                                                          SOL00080
      IF(NX.NE.0) GO TO 120                                             SOL00090
      CALL SOLVE(N,A,Y,X)                                               SOL00100
      RETURN                                                            SOL00110
  120 CONTINUE                                                          SOL00120
      IF(NY.NE.0)  GO TO 140                                            SOL00130
      DO 130  I=1,N                                                     SOL00140
      Y(I)= 0.                                                          SOL00150
      DO 130 J= 1,N                                                     SOL00160
  130 Y(I)=Y(I) +A(I,J)*X(J)                                            SOL00170
      RETURN                                                            SOL00180
  140 CONTINUE                                                          SOL00190
      DO 200 I= 1,NY                                                    SOL00200
      DO 200 J= 1,NY                                                    SOL00210
  200 B(I,J)=A(LISTY(I),LISTY(J))                                       SOL00220
      DO 300  I= 1,NY                                                   SOL00230
      C(I) = Y(LISTY(I))                                                SOL00240
      DO 300  J=1,NX                                                    SOL00250
  300 C(I)= C(I)-A(LISTY(I),LISTX(J))*X(LISTX(J))                       SOL00260
      CALL SOLVE(NY,B,C,Z)                                              SOL00270
      DO 400 I=1,NY                                                     SOL00280
  400 X(LISTY(I))= Z(I)                                                 SOL00290
      DO 800 I= 1,NX                                                    SOL00300
      C(I)=0.                                                           SOL00310
      DO 600  J= 1,NY                                                   SOL00320
  600 C(I)=C(I) +A(LISTX(I),LISTY(J))*X(LISTY(J))                       SOL00330
      DO 700 J= 1,NX                                                    SOL00340
  700 C(I) =C(I)+A(LISTX(I),LISTX(J))*X(LISTX(J))                       SOL00350
  800 Y(LISTX(I))= C(I)                                                 SOL00360
      RETURN                                                            SOL00370
      END                                                               SOL00380
                                                                        SOL00390
```

```
      SUBROUTINE SOLVE(N,A,C,X)                                          SOL00010
       DATA NWRITE/6/
      DIMENSION A(N,N),X(N),C(N)                                         SOL00020
      DO 400  K= 1,N                                                     SOL00030
      BIG = ABS(A(K,K))                                                  SOL00040
      IBIG = K                                                           SOL00050
      DO 100   I=K,N                                                     SOL00060
      SIZE = ABS(A(I,K))                                                 SOL00070
      IF(SIZE.LT.BIG)  GO TO 100                                         SOL00080
      BIG=SIZE                                                           SOL00090
      IBIG =I                                                            SOL00100
  100 CONTINUE                                                           SOL00110
      IF(K.EQ.IBIG) GO TO 280                                            SOL00120
      DO 200  J= K,N                                                     SOL00130
      ABIG = A(IBIG,J)                                                   SOL00140
      A(IBIG,J)=A(K,J)                                                   SOL00150
  200 A(K,J)= ABIG                                                       SOL00160
      CBIG =C(IBIG)                                                      SOL00170
      C(IBIG) =C(K)                                                      SOL00180
      C(K) =CBIG                                                         SOL00190
  280 CONTINUE                                                           SOL00200
      IF(A(K,K).EQ.0.) GO TO 600                                         SOL00210
      DO 350 I= 1,N                                                      SOL00220
      IF(I.EQ.K) GO TO 350                                               SOL00230
      RATIO = A(I,K)/A(K,K)                                              SOL00240
      DO 300  J=K,N                                                      SOL00250
      A(I,J)=A(I,J)-RATIO*A(K,J)                                         SOL00260
      CALL OVERFL(IQ)
      IF(IQ.EQ.2)GO TO 301
      WRIRE(6,98)
   98 FORMAT(1X,'IQ I       J     K  A(I,J)     A(K,J)                RATIO')
      WRITE(6,99)IQ,I,J,K,A(I,J),A(K,J),RATIO
   99 FORMAT(1X,4I5,3E15.6)
C     IF IQ= 1  OVERFLOW......IQ=3  UNDERFLOW
  301 CONTINUE
  300 CONTINUE
      C(I) = C(I) -RATIO*C(K)                                            SOL00270
  350 CONTINUE                                                           SOL00280
  400 CONTINUE                                                           SOL00290
      DO 500 K= 1,N                                                      SOL00300
  500 X(K) = C(K)/A(K,K)                                                 SOL00310
      RETURN                                                             SOL00320
  600 WRITE(6,666)                                                       SOL00330
  666 FORMAT(10X,'SINGULAR MATRIX')                                      SOL00340
      DO 700 I= 1,N                                                      SOL00350
  700 X(I)= 0.                                                           SOL00360
      WRITE(NWRITE,2)K
    2 FORMAT(6X,'K=',I5)
```

```
RETURN
END
```

SOL00380

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